COMMITMENT AND THE ADOPTION OF A COMMON CURRENCY*

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In contrast to Mundell’s inquiry on the optimality of currency areas, this article aims to understand under what circumstances a Pareto-dominant monetary union will be established. Using a multicountry overlapping generations model, we highlight gains from monetary union arising from reduced transactions costs and lower inflation. Despite these gains, countries acting independently will impose barriers to exchange through local currency restrictions, thereby creating transactions costs and providing an incentive for inflation. Therefore, the gains from monetary union are most likely to be lost without collective effort.

1. INTRODUCTION

The celebrated contribution of Mundell (1961) focuses on the appropriate domain of a currency area. According to his analysis, the determination of optimum currency areas reflects a trade-off between the ease of transactions associated with the use of a single currency and potential instability arising from the delegation of monetary control to a single authority.

This article, in contrast, tackles a different issue: the equilibrium domain of a currency area. We aim to understand under which conditions a common currency area will actually be created. Using a model in which there are, by construction, gains to the adoption of a common currency, we find that such gains may not be easily realized. Instead, we argue that countries, acting strategically, will have an incentive to deviate from a proposed common currency, thus destroying the mutual gains from cooperation as in the traditional prisoners’ dilemma game.

Our analysis has two principal components. First we construct a dynamic equilibrium model to formalize the gains to a common currency. Second, we discuss the interactions between national governments and the incentives to deviate from a monetary union.

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The model we study in this article indicates two key gains to common currency. First, the adoption of a single currency reduces the frictions associated with the flow of goods across countries. In our abstract formulation, these gains arise from the inability of agents to costlessly respond to specific changes in their tastes for goods in a multicurrency world (studied in Section 2 of the article).\(^2\) Second, our model highlights the price stability benefit of a common currency from the centralization of monetary policy. In a world with multiple independent currencies and local currency cash-in-advance constraints, governments maximize national welfare by creating excessive inflation. Although distortionary, inflation is desired, as it taxes the money holdings of foreign agents and thus benefits home citizens.\(^3\) In contrast, a central bank under common currency will internalize the interdependence between countries and optimally choose a lower inflation rate.

Although this argument is cast here through an abstract seignorage game between governments, there is a very general and powerful point underlying the analysis: the gains to centralization arise from the internalization of the external effects of national policies.\(^4\) Applied to a monetary setting, countries perceive a mutual advantage to the adoption of joint policies that eliminate seignorage. Yet, the absence of a commitment mechanism implies that countries, acting independently, are unable to avoid inflation.

The second component of the article concerns the reaping of these welfare gains: will individual countries acting independently adopt policies to exploit the gains to a common currency? To develop this theme, Section 4 of the article studies a game in which governments choose both their monetary institutions (whether or not to impose local currency requirements) and their inflation rates. We first argue that the allocation under a common currency is identical to that achieved in a world economy with local currencies as long as governments do not impose a local currency requirement and do not inflate. However, we show that, acting independently, governments will choose to deviate from the common currency outcome by imposing local currency requirements and inflating unless such deviations lead to a very strong “punishment” such as autarchy. That is, the common currency allocation is not generally a subgame-perfect Nash equilibrium of our game. We provide conditions such that the Nash equilibrium entails the imposition of local currency requirements and positive inflation. In this way, the paper contributes to the theoretical international macroeconomics literature by providing an explanation for local currency cash-in-advance constraints.

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\(^2\) As discussed in more detail below, our model does not have explicit transactions costs. Instead the inefficiencies appear as an ex post misallocation of consumption goods.

\(^3\) On that account, seignorage is similar to other policies that a government can use to influence a country’s terms of trade, like a tariff policy. For related discussion, see Canzoneri (1989) and Aizenman (1992).

\(^4\) This theme appears often in the literature on fiscal federalism and trade policies as well. Inman and Rubinfeld (1997) provide a recent discussion of these issues. Dixit (1987) discusses this point in the context of trade policies as devices for profit shifting. In reference to monetary policy, the European Commission report (Emerson et al., 1992, p. 114) notes: “... the adoption of a common monetary policy handled by EuroFed will remove the possibility of beggar-thy-neighbour monetary and exchange rate policies.”
Thus, if punishments from defecting from a common currency outcome are not too severe, the game between governments over international monetary arrangements is one of a prisoners’ dilemma. From this perspective, the adoption of a common currency serves as a commitment device, though the commitment problem we emphasize is between countries rather than the more common specification of an internal commitment problem between the monetary authority and private agents. Consequently, the adoption of a common currency must arise from the “cooperation” of both countries and requires joint action for its establishment and stability.

But, if punishments are severe enough, the choice of the international monetary regime is a coordination game. In this case, some coordination device, such as a treaty, may be needed to select the Pareto-superior, common currency, equilibrium.

2. WORLD ECONOMY WITH LOCAL CURRENCIES

We consider an overlapping generations structure in which all agents live for two periods. The horizon is infinite with time indexed by \( t = 1, 2, \ldots \). Further, there are two countries, “home” and “foreign,” which are identical. In each economy there is a continuum of consumers subject to idiosyncratic shocks. There is trade across these countries (explained in detail below) but labor is immobile. Each country’s government issues its local currency and requires that local goods are traded through the use of this currency. This corresponds to the imposition by each government of a local currency (cash-in-advance, hereafter CIA) constraint. 5

2.1. Optimization and Market Clearing. The analysis will first focus on the optimization problem of a representative agent in the home country. The next subsection looks at market clearing. We then characterize the monetary steady state. The main proposition of this section shows that equilibrium inflation rates are positive.

2.1.1. Optimization. The optimization problem of a representative, generation \( t \) home agent is given by

\[
\max_{m_h, m_f, n, \epsilon [0,1]} E_\theta (\theta \ln(c_{t+1}^h) + (1 - \theta) \ln(c_{t+1}^f)) - g(n_t)
\]

5 This approach is common in the open economy macroeconomics literature; see Obstfeld and Rogoff (1997) and Sargent (1987, Ch. 5). We adopt a legal restrictions perspective, in the spirit of Bryant and Wallace (1984), and view the government as having the right to impose constraints on the nature of transactions. In fact, it is precisely this power that allows a government to negotiate the adoption of a common currency. Given that power, we find that indeed governments have an incentive to impose CIA constraints.

Recent papers have sought to justify the local cash-in-advance constraints by means of search theories of money, as in Kiyotaki et al. (1993), for example. This search theoretic approach certainly highlights coordination and the gains to a common currency but analyzing government interventions in that type of framework is more difficult.
subject to

\[ p_t n_t = m^h_t + e_t m^f_t \]

and

\[ c^h_{t+1} = \left( m^h_t + \tau_{t+1} \right) / p_{t+1} \]
\[ c^f_{t+1} = m^f_t / p^*_{t+1} \]

Old age utility from consumption is a sum of two terms, \( \theta_t \ln(c^h_{t+1}) + (1 - \theta_t) \ln(c^f_{t+1}) \), where \( c^j_{t+1} \) is the consumption of good \( j \in \{h, f\} \) in period \( t + 1 \). The level of work, given by \( n_t \), is between zero and one as each agent has a unit endowment of time. The disutility of work is represented by \( g(n_t) \), assumed to be increasing, convex, and continuously differentiable. For simplicity, we assume that one unit of output is produced with one unit of labor. The variables \( p_t \) and \( p^*_t \) are the prices of goods \( h \) and \( f \), denominated in home currency and foreign currency, respectively.

The random variable \( \theta \) represents a shock to the tastes of the agent. It is critical that \( \theta \) is realized at the start of old age and thus after the choices of employment and currency holdings. We assume that \( \theta \in [0, 1] \) and all agents in both countries draw from the same distribution given by \( H(\theta) \), with mean \( \bar{\theta} \). Realizations of this shock are independent across agents and countries.\(^6\)

The first constraint implies that an agent takes money earned in youth and allocates it to the holdings of domestic currency \( (m^h_t) \) and foreign currency \( (m^f_t) \); \( e_t \) denotes the period \( t \) exchange rate. The second and third constraints correspond to the cash-in-advance constraints faced by old agents and reflect two important aspects of this world economy. First, within a period, exchange markets open after goods markets. Hence, old agents cannot adjust their portfolio holdings before going to goods markets: this is a basic friction in our model.\(^7\) Second, agents are required to make purchases using local currency. Presently this is an assumption, though in Section 4 we show that this type of constraint will arise endogenously. The combination of both assumptions generates the existence of cash-in-advance constraints: an agent needs to hold both currencies at the start of his old-age period in order to consume both goods. The suppression of one of these two assumptions would eliminate this constraint.

The process of money creation is reflected in the evolution of domestic money holdings. In particular, each generation \( t \) agent, regardless of his money holdings, receives a transfer of \( \tau_{t+1} \) at the start of old age. This transfer is perfectly anticipated. Note that an agent does not receive transfers of foreign currency: this is the basis of the inflation tax on foreign currency holders which, as we shall see, redistributes real wealth to domestic citizens.

\(^6\) For simplicity, we omit the \( i \) and \( t + 1 \) subscripts in the taste shock.

\(^7\) This is a broader view of market frictions than simply imposing a trading cost. Adding such a cost to our model would modify our analysis since some agents would, ex post, elect to pay this cost to modify their portfolio of currencies. In effect, this is similar to reducing the variability of the taste shock in our present formulation.
The first-order conditions for the agent’s optimization problem are summarized by two conditions,

\[ g'(n_t) = \frac{p_t}{p_{t+1}} \bar{\theta} c_{t+1}^{h} \quad \bar{\theta} = \frac{c_{t+1}^{h}}{1 - \theta} \frac{e_t p_{t+1}^{h}}{p_{t+1}} \]

The first equality relates the marginal disutility of work to the marginal utility from the consumption of the home good. Note that optimal expenditure shares, given by the second equality, are based on $\bar{\theta}$ and not on the realization of this variable since portfolio decisions are made ex ante.\(^8\)

2.1.2. Market clearing. In each period, five markets must clear: the two goods markets, the two money markets, and the exchange market. The goods markets open at the start of the period. Old agents from both countries use their money holdings to purchase the corresponding goods produced by young agents. Each national money market is the flip side of these transactions: young agents sell their goods to acquire the local currency held by the old. Finally, in the exchange market, currencies are exchanged as young agents optimally set their portfolios.

In the home country, money market clearing is given by

\[ M_t = p_t n_t \]

The left side of (4) is the exogenously given stock of fiat money per (home) capita and the right side is the money value of output per home producer in period $t$. The evolution of the stock of home money is given by

\[ M_{t+1} = M_t (1 + \sigma) \]

where $\sigma$ is the fixed growth rate of money in the home economy, later to be chosen by the government. The money supply increase finances the lump-sum transfers received by home residents,

\[ \tau_{t+1} = M_t \sigma \]

The condition for exchange market equilibrium is

\[ m_{t}^{sh} = e_t m_t^{f} \]

The left side of this condition is the per capita money holdings of home currency by young foreigners. The right side is the value, in terms of home country units of account, of the foreign money holdings of young home agents. Both quantities

\(^8\) Unless stated otherwise, throughout the article, equations for the foreign country are to parallel those for the home country and therefore will not be explicited.
must be of equal value. Finally, the exogenous stocks of domestic and foreign currencies must be held by someone after the close of the exchange markets,

\[ M_t = m^h_t + m^{*h}_t \quad \text{and} \quad M^{*}_t = m^{*f}_t + m^f_t \]

2.2. Equilibrium. Given the conditions for optimization by the representative agent in each country and the market clearing conditions, we first characterize the steady-state equilibrium with valued fiat money in both economies given the two exogenous rates of money growth, \( \sigma \) and \( \sigma^* \). We then turn to the determination of the equilibrium rates of inflation.

2.2.1. Steady states given rates of money creation. Let \( n \) and \( n^* \) represent the steady-state employment levels in the two countries and let agents in the home (foreign) country hold a fraction \( \phi (\phi^*) \) of their local currency earnings in youth in domestic currency. A steady-state monetary equilibrium is given by \( (\phi, n, \phi^*, n^*) \) and a price system \( (p_t, p^*_t, e_t)_{t=1}^{\infty} \) such that the conditions for individual optimization and market clearing are met. Focusing on interior monetary equilibria in which fiat money is valued in both economies, our characterization of the steady state is summarized by:

**Proposition 1.** For every \( \sigma \in (-1, 1/Z), \sigma^* \in (-1, 1/Z) \), there exists a unique, interior monetary steady state characterized by

\[ \phi = \frac{1 - Z\sigma}{1 + Z} \quad \phi^* = \frac{1 - Z\sigma^*}{1 + Z} \]

\[ \frac{1}{1 + \sigma} = ng'(n) \quad \frac{1}{1 + \sigma^*} = n^*g'(n^*) \]

where \( Z \equiv (1 - \bar{\theta})/\bar{\theta} \).

**Proof.** See Appendix.

There are some interesting properties of the steady state. First, in the usual manner, inflation brought about by lump-sum transfers creates a tax on domestic productive activity. Hence, an increase in the rate of domestic inflation reduces employment and output.

Second, domestic output and employment do not depend on the foreign rate of money creation. In the same way, portfolio shares do not depend on foreign rates of money creation. These simplifications, of course, reflect the assumption

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9 Due to the Cobb–Douglas specification of the utility function, there is a unique monetary steady state and no nonstationary equilibria. As is customary in overlapping generations models there will also be an autarchic equilibrium where money has no value. In an autarchic equilibrium, consumption of all goods by all generations is zero, as is the labor input. Expected utility is therefore \(-\infty\). This equilibrium will come into play in Section 4.
of Cobb–Douglas preferences with the resulting implications for constant budget shares.\textsuperscript{10}

Third, using these conditions, the steady-state equilibrium levels of consumption by home agents are given by

\begin{equation}
\begin{aligned}
ch &= \frac{(\phi + \sigma)}{(1 + \sigma)} n(\sigma), \\
ct &= \frac{(1 - \phi^*)}{(1 + \sigma^*)} n^*(\sigma^*)
\end{aligned}
\end{equation}

Note that the rate of money creation in foreign countries does influence the utility level of home agents through their equilibrium consumption levels of foreign goods. Further, in equilibrium, domestic money creation has three apparent influences: directly on the level of employment, through the transfer (the numerator of \(ch\)), and through the rate of price inflation (the denominator of \(ch\)).

\subsection*{2.2.2. Determination of equilibrium inflation rates.}

Using the equilibrium from Proposition 1, let \(V(\sigma, \sigma^*)\) and \(V^*(\sigma^*, \sigma)\) be the lifetime expected welfare of an agent in the home and foreign countries respectively. Formally,

\[
V(\sigma, \sigma^*) = E_\theta(\theta \ln(ch) + (1 - \theta) \ln(ct)) - g(n(\sigma))
\]

and

\[
V^*(\sigma^*, \sigma) = E_\theta(\theta \ln(c^*f) + (1 - \theta) \ln(c^*h)) - g(n^*(\sigma^*))
\]

where the functions \(n(\sigma)\) and \(n^*(\sigma^*)\) are determined in the equilibrium established in Proposition 1 and the consumption levels are given above. Since \(g(n)\) is assumed to be continuously differentiable, \(n(\sigma)\) and \(n^*(\sigma^*)\) are continuously differentiable functions of the money growth rates.

We restrict the government to choose the rate of inflation at the start of time. The objective function of the government is to maximize the lifetime expected utility of a representative agent. Given our focus on steady states, this representative agent could be in any generation.\textsuperscript{11} The gains and losses from variations in the rate of inflation are evident from (8) and (9). Given the employment levels, \(n\) and \(n^*\), home inflation increases the state contingent consumption of home agents whereas foreign inflation reduces it. From the definition of \(ch\), in order for home consumption to increase with \(\sigma\), \(\phi\) must be less than one: i.e., not all of the domestic money supply is held by domestic citizens. In this economy, agents hold the currency of the other country in order to finance their consumption in old age. The local currency requirement creates a demand for local currency by foreign agents.

\textsuperscript{10} Note that this implies that currency substitution effects are entirely absent from this model.

\textsuperscript{11} In fact, our results are more general. From (5) and (9), it is only the period \(t + 1\) level of money creation that influences the lifetime utility of generation \(t\). Past money creation has no real effects. Thus, there is really no “state variable” inherited from the past that matters for real allocations. So, we can focus on the maximization problem generation by generation without loss of generality.
and thus a basis for the inflation tax. This can be seen directly from the characterization of $c^h$: if $\phi = 1$, then the only effect of $\sigma$ would be to distort the labor supply decision.

Of course, the cost of inflation arises from the distortions it creates. Higher home inflation reduces the incentive for home agents to produce, which ultimately reduces $c^h$ and thus lifetime utility. Still, starting at zero inflation, there is an incentive for countries to increase their money supplies. This observation leads to

**Proposition 2.** The symmetric equilibrium of the game between governments entails positive money growth rates: $\sigma = \sigma^* = Z$.

**Proof.** See Appendix.

In fact, the equilibrium with positive inflation is a dominant strategy equilibrium. Even though the foreign rate of inflation has a (negative) effect on the welfare of home agents, it has no influence on the optimal level of domestic inflation.

Note that this model produces inflation without resorting to the assumption of a commitment problem between the government and private agents within a country, as in the well-known literature stemming from Kydland and Prescott (1977). Instead, the incentive to inflate arises from the interaction across countries induced by the pattern of money demand. Thus the inflation that emerges in the equilibrium with multiple currencies requires joint action, such as the adoption of a common currency.

### 3. COMMON CURRENCY

This section argues that the creation of a monetary arrangement in which there is a single currency and a single central bank will lead to a welfare increase for all agents. These welfare gains come from the increased liquidity created by a single currency and by the elimination of the incentive for unilateral inflation, which, from Proposition 2, arises in a world economy with multiple currencies.

**3.1. The Common Currency World Economy.** Let $M_t^{CC}$ represent the stock of common currency in period $t$, $\sigma^{CC}$ the growth rate of this stock, and $q_h^t(q_f^t)$

12 Stated differently, each country has an incentive to inflate in order to reduce its own output and thus influence terms of trade. Certainly, there are many ways in which countries can influence terms of trade. But, through European and world-wide negotiations on relaxing trade restrictions, the policies available for this purpose have certainly narrowed. From this perspective, using the inflation tax becomes a progressively more important tool. Although the model, through its specification of a Cobb-Douglas structure, might certainly limit currency substitution and so may overstate the power of this tool, the “abuse” of this inflation tax seems to us to be the most likely motivation for a collective agreement on monetary arrangements. That is, the other “beggar-thy-neighbor” policies may be dealt with through agreements on real not nominal variables. The European Commission also notes this point as we discussed in footnote 4. In fact, an inflation tax can replace a proportional labor tax as a means to influence terms of trade.
the period $t$ money price of home (foreign) goods. The optimization problem of a representation agent of generation $t$ in the home country is given by

$$\max_{c^h_{t+1}, c^f_{t+1}, n_t \epsilon [0,1]} \mathbb{E}_{\theta} \{ \theta \ln(c^h_{t+1}(\theta)) + (1 - \theta) \ln(c^f_{t+1}(\theta)) \} - g(n_t)$$

subject to

$$c^h_{t+1}(\theta) q^h_{t+1} + c^f_{t+1}(\theta) q^f_{t+1} = q^h_t n_t + \tau^{CC}_{t+1} \equiv I_t \text{ for all } \theta$$

In contrast to the problem specified in the previous section, (1)–(2), here there is a single budget constraint for each value of $\theta$ since, in period $t+1$, the agent will take money earned in youth plus a transfer of $\tau^{CC}_{t+1}$ and purchase both home and foreign goods after observing the taste shock. Ex post, agents will respond to variations in tastes with demands given by

$$c^h_{t+1} = \theta I_t / q^h_{t+1} \text{ and } c^f_{t+1} = (1 - \theta) I_t / q^f_{t+1}$$

Given this allocation of income, agents choose labor supply to solve (10) yielding

$$\frac{1}{1 + \sigma^{CC}} = ng'(n)$$

3.2. Equilibrium and Optimal Inflation. We assume that newly created currency is distributed equally across all agents. Let $n^{CC}(\sigma^{CC})$ denote the steady-state value of employment for both home and, by symmetry, foreign agents, and $\sigma^{CC}$ the steady-state rate of money growth. The steady-state level of utility can be expressed as

$$V^{CC}(\sigma^{CC}) = \mathbb{E}_{\theta} \{ \theta \ln(\theta n^{CC}(\sigma^{CC})) + (1 - \theta) \ln((1 - \theta) n^{CC}(\sigma^{CC})) \} - g(n^{CC}(\sigma^{CC}))$$

Then we can state

**Proposition 3.** The optimal monetary policy in a common currency area is $\sigma^{CC} = 0$.

**Proof.** See Appendix.

So, the common central bank will optimally choose zero inflation and the common currency implies that agents can, ex post, optimally respond to their taste shocks. The optimality of zero inflation reflects the internalization of the “beggar they neighbor” policies that led to inflation in the multiple currency world economy.14

13 Clearly then an interesting extension is to study political power with regard to the creation and distribution of newly created money. See, for example, Casella and Feinstein (1991) and Chang (1995).

14 This corresponds to the Friedman rule, since the real rate of return is zero in this economy.
3.3. Measuring the Gains to a Common Currency. Let $V^{LC}$ be the expected utility of an agent in the equilibrium of the world economy with local currencies given that both countries inflate at rate $Z$. Using Proposition 2, this is given by

$$V^{LC} = E_\theta \left\{ \theta \ln(1/(1 + Z)) + \ln(n(Z)) + (1 - \theta) \ln(Z/(1 + Z)) \right\} - g(n(Z))$$

where $n(Z)$ is the steady-state equilibrium level of employment given that the rate of money creation is $Z$.

In a similar manner, Proposition 3 implies that the lifetime utility in a common currency area ($V^{CC}$) is given by

$$V^{CC} = E_\theta \left\{ \theta \ln(\theta) + \ln(n(0)) + (1 - \theta) \ln(1 - \theta) \right\} - g(n(0))$$

where $n(0)$ is the steady-state level of employment at zero inflation.

Using these expressions, let $\Delta(Z) \equiv V^{CC} - V^{LC}$ be the utility gain from a common currency area

$$\Delta(Z) = E_\theta \left\{ \theta \ln(\theta) + (1 - \theta) \ln(1 - \theta) \right\} + \left[ \ln(n(0)) - g(n(0)) \right] - \left[ \ln(n(Z)) - g(n(Z)) \right]$$

Both terms in this expression are positive and related to increased efficiency and price stability.

The first term represents the gain associated with achieving an ex post efficient allocation of goods, given the uncertainty in tastes. If there was no uncertainty, so that $\theta = \bar{\theta}$, then it would be zero. Using a second-order Taylor series expansion, it is approximately equal to

$$E_\theta \left\{ \theta \ln(\theta) + (1 - \theta) \ln(1 - \theta) \right\} = \frac{1}{2} \text{var}(\theta) \left( \frac{1}{\bar{\theta}} + \frac{1}{(1 - \bar{\theta})} \right)$$

This gain from common currency is proportional to the variance in the taste shocks since the single currency regime allows agents to respond to taste shocks.

The second term in $\Delta(Z)$ represents the gain from common currency associated with the reduction in the rate of inflation from $Z$ to 0. Since inflation is distortionary, this gain is related to the responsiveness of the labor supply decision to the rate of inflation and the magnitude of $Z$. A second-order Taylor series expansion of the second term in (13) implies

$$\left[ \ln(n(0)) - g(n(0)) \right] - \left[ \ln(n(Z)) - g(n(Z)) \right] \approx \frac{1}{2} \left[ \frac{n(Z) - n(0)}{n(0)} \right]^2 \left( 1 + \frac{g''(n(0))n(0)}{g'(n(0))} \right) > 0$$

The first term on the right measures the gap between employment in the local currency regime and that arising under a common currency area. The second term
on the right reflects the curvature in the disutility of work function: the larger the curvature the larger is the loss in utility due to inflation.\textsuperscript{15}

4. ADOPTING A COMMON CURRENCY

Admittedly, the present model gives a very favorable view of a common currency, as no costs are associated with a single currency and a single monetary authority. But we claim that even in this most favorable setting, the adoption of a common currency between sovereign countries requires a very strong international commitment, as recently exemplified by the lengthy and painful negotiations leading to the European common currency. It is most unlikely that the gains associated with a single currency will come from uncoordinated government decisions.

We study this issue by constructing a multistage game between governments in which the monetary regime emerges endogenously. In the first stage, governments choose their monetary regime, either imposing a local currency cash-in-advance constraint ($LC$) or not ($NLC$).\textsuperscript{16} If they choose not to impose a local currency, then any currency can be used for making purchases in that economy. In the second stage, governments simultaneously choose a rate of inflation for their money supply. Finally, we look at a steady state of the world economy given these policy choices.

Formally, let $r \in \{LC, NLC\}$, $r^* \in \{LC^*, NLC^*\}$ denote the first-stage strategy of the home (foreign) country. The second-stage strategy of the home (foreign) government is then $\sigma(R)(\sigma^*(R))$ where $R = (r, r^*)$. Let $\Xi(R) \equiv (\sigma(R), \sigma^*(R))$. Given $(R, \Xi(R))$, a competitive steady-state equilibrium of the overlapping generations model is determined. Let $\xi(R, \Xi(R))$ be the set of steady-state competitive equilibria. The construction of a subgame-perfect Nash equilibrium requires the selection of a steady state from this set.

4.1. Is the Common Currency Area Allocation a Nash Equilibrium? First, we establish that the allocation with a common currency is possible in this extensive form game. We then determine whether or not it is a Nash equilibrium of the game.

4.1.1. Is the common currency area allocation feasible? We begin our analysis of this game by relating the choice of monetary regime and the rate of inflation to a common currency. This is important since this result implies that the countries, acting independently, could achieve the common currency area allocation characterized by a single currency and zero inflation.

\textsuperscript{15}For example, if $g(n) = n/k$, where $k$ is a constant, then $n(Z) = k/(1 + Z)$ and (15) simplifies to $[\ln(n(0)) - g(n(0))] - [\ln(n(Z)) - g(n(Z))] = k^2(1 - \bar{\theta}) - \ln(\bar{\theta})$. So, if countries did not trade ($\bar{\theta} = 1$), then this term would be zero because the level of inflation would be 0. As $\bar{\theta}$ falls from 1, this term increases, reflecting the difference between $n(0)$ and $n(Z)$.

\textsuperscript{16}Consideration of this economy without local currency constraints, which is a generalization to the two good case of Kareken and Wallace (1981), requires modification of the basic model presented in Section 2. The characterization of this economy is contained in the appendix.
PROPOSITION 4. The common currency area allocation is an element of \( \xi(NLC, NLC^*, 0, 0) \).

PROOF. See Appendix.

The intuition for this result is relatively straightforward. In an equilibrium in which both currencies have value, their rates of return must be equal. Since neither country imposes an LC requirement, agents can respond to their realized taste shocks even though they could have a single currency in their portfolio. Given that neither country inflates by assumption, the labor supply decisions for both home and foreign agents are efficient as in the proof of Proposition 2. From this, we see immediately that the resulting allocation is the same as that obtained under common currency.

To be clear, the point of this proposition is only that the common currency allocation is a possible outcome of the interaction between countries. In this way, studying the outcome of the monetary regime game sheds light on common currency. The proposition does not state that if neither country imposes an LC constraint, then the common currency area outcome obtains.

4.1.2. Defections from a common currency. The main result of this section is that the configuration of actions in which neither country imposes a local currency requirement is not a subgame-perfect Nash equilibrium. This result is important for two reasons. First, it indicates that common currency will not be an equilibrium of the monetary regime game since countries have unilateral incentives to impose LC constraints. Second, this result justifies the imposition of LC constraints by governments interested in maximizing the expected utility of their own agents, a constraint that was imposed in the economy we considered in Section 2.17

For this argument, we must consider the implications of defection by one of the governments from the candidate equilibrium. The requirement of subgame perfection implies that the continuation payoffs be evaluated at an equilibrium of the overlapping generations model given the policy choices of the government, i.e., these payoffs are calculated from an element of \( \xi(R, \Xi(R)) \). However, the overlapping generations model has many equilibria. As we will see, this freedom of selecting an equilibrium enables us to argue that the common currency allocation is not subgame perfect by constructing continuation payoffs that provide higher welfare than that achieved under the common currency allocation.

PROPOSITION 5. Assume that the selection from \( \xi(LC, NLC^*, \sigma, \sigma^*) \) is the competitive equilibrium in which only the home currency has value; then the common currency allocation is not a subgame-perfect Nash equilibrium.

PROOF. See Appendix.

17 This result is in the same spirit as the Bryant and Wallace (1984) argument about legal restrictions providing a basis for price discrimination by a government.
The key to this proposition is the selection of a steady-state equilibrium in which only the currency of the single country that chooses $LC$ has value. Since this selection is from $\xi(LC, NLC^*, \sigma, \sigma^*)$, the policy configuration of $(NLC, NLC^*, 0, 0)$ is not a subgame-perfect Nash equilibrium. This selection seems quite reasonable since the currency of the country imposing the $LC$ requirement can be used in both countries whereas the other currency is only accepted in one country. Further, this selection seems intuitive in that a government would never defect from the proposed common currency allocation unless it thought it could increase the welfare of its citizens by doing so. Through our selection of an equilibrium, the deviating country’s currency is the one that has value and the foreign one has no value. The fact that the foreign country is not imposing a CIA is important since its agents can still consume. More generally, we are trying to distinguish the choice of monetary institution (CIA or no CIA) from the equilibrium that arises given that structure.

There are, however, other equilibria in the event that only one country imposes an $LC$ constraint. Suppose that autarchy is the competitive equilibrium that is selected in the event that either government defects from the common currency allocation and imposes a local currency requirement. As autarchy yields lower expected utility for the representative agent than common currency, neither government will have an incentive to impose a local currency requirement. This takes us part way to supporting common currency. But, will governments optimally choose not to inflate? We find

**Proposition 6.** Assume the selection from $\xi(LC, NLC^*, \sigma, \sigma^*)$ or $\xi(NLC, LC^*, \sigma, \sigma^*)$ for any $(\sigma, \sigma^*)$ is autarchy; then the common currency allocation is not a subgame-perfect Nash equilibrium.

**Proof.** See Appendix.

Evidently the threat of a reversion to autarchy is sufficient to dissuade governments from imposing local currency requirements, but does not create an incentive for zero inflation. Even if both currencies are used without any local currency restrictions, we find that governments will have an incentive to inflate. Essentially, the ability to make nominal transfers to their own citizens provides governments with a means of increasing welfare even when both currencies are held in equilibrium. Hence, the common currency allocation is not an equilibrium.

4.1.3. **Supporting the common currency as a subgame-perfect Nash equilibrium.** These results are reversed when we assume that any deviation from the common currency allocation will be followed by the autarchic competitive equilibrium in which neither currency has value. That is, if governments choose $(NLC, NLC^*, 0, 0)$, then the resulting competitive equilibrium is simply the stationary interior monetary equilibrium characterized in Propositions 1 and 2. Else, the competitive equilibrium is autarchy.

**Proposition 7.** Assume the selection from $\xi(LC, NLC^*, \sigma, \sigma^*)$, $\xi(NLC, LC^*, \sigma, \sigma^*)$ for any $(\sigma, \sigma^*)$ is autarchy. Further assume that the selection from $\xi(NLC,$
\[ NLC^*, \sigma, \sigma^* \) for \((\sigma, \sigma^*) \neq (0, 0) \) is also autarchy. Then, the common currency area allocation is a subgame-perfect Nash equilibrium.

\textbf{PROOF.} See Appendix.

Clearly, the distinguishing feature between this result and those reported in Propositions 5 and 6 is that private agents assign zero value to the currency of any government that defects from the common currency allocation. This defection may either entail the imposition of a local currency requirement and/or money creation. This path following the defection by a government from a candidate equilibrium is in fact one of the many equilibria of the overlapping generations model. Thus, common currency can be supported as a subgame-perfect Nash equilibrium. Note though that the “punishment” of autarchy arising from inflation and/or the imposition of local currency requirements might be viewed as rather severe and thus this equilibrium seems less plausible than those characterized by Propositions 5 and 6.

4.2. *Is LC a Nash Equilibrium?* We now study conditions such that the imposition of local currency requirements by both countries is a subgame-perfect Nash equilibrium. This is relevant to our understanding of the incentives because countries have to erect barriers to exchange thereby forbidding the common currency allocation.

\textbf{Proposition 8.} \textit{If the variance of } \theta \textit{is sufficiently small, then } (LC, LC^*, Z, Z^*) \textit{is a subgame-perfect Nash equilibrium.}

\textbf{Proof.} \textit{See Appendix.}

Note that this result requires that the variance of taste shocks be small. The gain to imposing a local currency requirement is the creation of an inflation tax base. The gain to a common currency increases with uncertainty in tastes. Therefore, if the variance of these shocks is sufficiently small, then a country should prefer to use its own currency as a shield against the inflation of another country.\(^{18}\)

Recall from Proposition 2 that the choice of \(\sigma = Z(\sigma^* = Z^*)\) was a dominant strategy. Thus we can see that our requirement that governments choose the rate of money creation once at the start of time is not restrictive. If we allowed our governments to choose the rate of money creation in each period, Proposition 8 would still hold.

5. **Conclusion**

Is cooperative action needed to reap the gains to a common currency? Our answer to this question depends on the selection of a competitive equilibrium given defections from the common currency allocation. Clearly, if the resulting

\(^{18}\) This is essentially the point made by Fischer (1982) concerning the gains to a national currency.
“punishment” by the market is severe enough, then the gains of common currency will arise simply from countries independently eliminating barriers to exchange and the inflation tax. To the extent that this type of equilibrium selection is not credible, then Propositions 5 and 6 imply that the gains to a common currency are lost without a cooperative effort on the part of governments (the electorate in the case of democracies).

From this perspective, the adoption of a common currency may proceed by the design of an institution in which (i) there is a common currency, (ii) the quantity of this currency is controlled by a single central bank, and (iii) citizens of countries outside the common currency area are required to use the common currency for purchases from member countries. This is precisely the model of a common currency area, with two countries, studied in Section 3. Given the presence of this institution, reflecting the cooperative efforts of governments, each government may then be called upon to independently choose whether to join the common currency area or not. Under the proviso that a common currency area with a single country is the same as a single country with its own currency, clearly joining the common currency area is a dominant strategy.

Thus, we find that indeed the gains to the common currency area may be realized once countries come together to set the appropriate terms for this institution. But, absent this collective effort, these gains are most likely to be lost as governments pursue the self-interest of their citizens and thus erect barriers to exchange and inflate.

With respect to the recent establishment of the European Monetary Union, we have undoubtedly failed to capture some of the apparent differences across countries in Europe given our assumed symmetric environment. Clearly the inflation experience of Germany differs from that of, say, Italy. Within the model, this might be captured by differences in the nature of taste shocks so that each country has a different value of $Z$. Hence, from Proposition 2, they would have (we suspect) different money creation rates. Or, outside of the model, Germany may have “enforced” an objective for its central bank to pursue “price stability” rather than the maximization of expected utility, as in our structure. Nonetheless, all countries would still gain in a monetary union.

Given these gains, our model and results provide an interesting perspective on the discussions among Western European countries that led to the creation of EMU and the euro. In combination with Propositions 5 and 6, Proposition 8 implies that the strategic interaction of adopting a common currency has a prisoners’ dilemma structure: there is a unique Nash equilibrium that delivers to each country a lower welfare than is obtained by cooperation. From this perspective, a treaty such as the Maastricht Treaty serves to support a cooperative outcome.

Alternatively, in combination with Proposition 7, Proposition 8 implies that the strategic interaction between governments has the structure of a coordination game with multiple Pareto ranked equilibria. From this perspective, the negotiations leading to the formation of the EMU played the role of selecting from a set of equilibria.

There are a number of natural extensions of our analysis. First, extending the analysis to a multicountry world seems feasible and desirable. This case would
allow a more interesting analysis of the game in which countries choose to join a common currency area or not, since, in contrast to the two country case, the defection of one country does not destroy the union.

Second, in characterizing these (perhaps modest) gains to common currency area, this article has avoided some potential costs of this institution. Returning to the analysis of Mundell, our model clearly lacks various sources of aggregate uncertainty that may give rise to the necessity of more complex policy stances and assumes a very strong centralized monetary authority. Relaxing these constraints would allow us to have a more balanced view of the desirability of a common currency area but would not modify the basic results of this article on the need for cooperation to support a desired common currency allocation.

Finally, the model can be extended to study dollarization: a regime in which only a single country imposes an \( LC \) constraint. Proposition 8 argued that this was not an equilibrium if taste shocks were not highly volatile. However, if tastes are variable enough, then the equilibrium may entail a form of dollarization in which only a single country imposes an \( LC \) constraint. In this case, the game between governments appears to have a Battle of the Sexes structure. Understanding dollarization merits further work.

\section*{APPENDIX}

\subsection*{A.1. Proofs}

\textbf{Proof of Proposition 1.} Here we focus on the conditions for the home country. Those for the foreign country are completely analogous. We use the conjectured portfolio shares along with the condition relating transfers to the stock of money, (5), to find

\begin{align}
\text{(A.1)} 
  p_t n_t - m^h_t &= (1 - \phi) M_t \\
\text{(A.2)} 
  m^h_t + t_{t+1} &= (\phi + \sigma) M_t
\end{align}

Substituting this into the second of the first-order conditions for the home country implies

\[ \frac{(1 - \phi)}{\phi + \sigma} = Z \]

which yields the first of the steady-state conditions after some manipulation.

\begin{itemize}
  \item[19] Of course, we do not claim that there are no other gains to monetary union. However, the gains we have identified do coincide with those stressed by the European Commission in its assessment of the EMU.
  \item[20] On the design of monetary union in an environment with both costs and benefits, see Chari and Kehoe (1998), Cooper and Kempf (2000), and Sibert (1992).
  \item[21] This case more accurately refers to “unofficial dollarization,” since the country’s authorities do not forbid the use of a foreign currency, while retaining their own currency. This is in contrast with “official dollarization” where the government authorities request that all trades be paid in a foreign currency, as has just been decided by Ecuador in 2000.
  \item[22] In Cooper and Kempf (2001), we develop an OLG model where dollarization arises as a way to solve seignorage games being played within a country.
\end{itemize}
Using the home market clearing condition as well as (A.1) in the first of the first-order conditions gives

\[
\frac{(1 - \bar{\theta})}{(1 - \phi)} = ng'(n)
\]

Substituting for \(\phi\) and using the definition of \(Z\) yields the second of the steady-state conditions for the home country. The bounds on these rates of money creation in the proposition guarantee that both \(\phi\) and \(\phi^*\) lie in the interval \((0, 1)\), which is necessary for an interior monetary equilibrium.

**Proof of Proposition 2.** In a steady-state equilibrium,

\[
V(\sigma, \sigma^*) = E_0 \left\{ \theta \ln(1/(1 + Z)) + \theta \ln(n(\sigma)) + (1 - \theta) \ln \left( \frac{(1 - \phi^*)^n \sigma(\sigma^*)}{1 + \sigma^*} \right) \right\} - g(n(\sigma))
\]

The home government maximizes this with respect to \(\sigma\), taking \(\sigma^*\) as given. This derivative equals zero when

\[
g'\left(\sigma^*(n)\right) = \bar{\theta}
\]

Using the condition characterizing \(n\) in the steady state (8) along with the definition of \(Z\) implies \(\sigma = Z\). A symmetric argument holds for the foreign country.

**Proof of Proposition 3.** The monetary authority solves \(\max_{\sigma^{CC}} V^{CC}(\sigma^{CC})\). This leads to

\[
1 = ng'(n)
\]

Using (12), this condition is met with \(\sigma^{CC} = 0\).

**Proof of Proposition 4.** This economy is a special case of one with two currencies, no local currency constraints, and governments each choosing a rate of money creation. Analysis of this economy is in Appendix Section A.2. The steady-state equilibrium in that economy with zero money creation for both currencies supports the common currency area allocation. Formally, setting \(\sigma^W = 0\) in the Appendix Section A.2 model yields the common currency area allocation. Essentially, there is no inflation tax and agents can costlessly respond to their taste shocks.

**Proof of Proposition 5.** Consider the policy configuration in which neither country imposes an LC requirement and then each chooses its inflation rate. The payoff from this cannot exceed that obtained under a common currency area, \(V^{CC}\), since the common currency area inflation rate was jointly optimal. Further,
from Proposition 4, we know that if neither country imposes an LC requirement and each sets the rate of money creation to zero, this is the same outcome as that obtained in a common currency area.

Suppose then that the home country defects from this proposed equilibrium and establishes an LC constraint. We focus on the steady-state equilibrium in which the home currency is the only one in circulation. Given this selection, the payoffs to a representative home agent are

\[
V_{L^C}(\sigma, 0) = E_\theta \{ \theta \ln(c^h(\theta, \sigma)) + (1 - \theta) \ln(c^f(\theta, \sigma)) \} - g(n(\sigma))
\]

where

\[
c^h(\theta, \sigma) = \frac{\theta n(\sigma)(\phi + \sigma)}{1 + \sigma}\phi \\
c^f(\theta, \sigma) = \frac{(1 - \theta)n^*(\phi + \sigma)}{(1 + \sigma)(1 - \phi)}
\]

Here \(\phi\) is again the share of the total money held by home agents and is given by

\[
\phi = \frac{\bar{\theta}\sigma + (1 - \bar{\theta})}{\sigma + 2(1 - \bar{\theta})}
\]

Further, \(n(\sigma)\) solves the representative agent’s first-order condition in the steady-state equilibrium,

\[
ng'(n) = \frac{\phi}{\phi + \sigma}
\]

Finally,

\[
n^*g'(n^*) = 1
\]

implying that the foreign agents are optimally choosing employment as well.

This is the same payoff level as that obtained in a common currency area if the rate of home money creation is zero: i.e., \(V_{L^C}(0, 0) = V_{CC}\). However, the derivative of \(V_{L^C}\) with respect to \(\sigma\) evaluated at \(\sigma = 0\) is given by

\[
1 - \bar{\theta} + 1 - \frac{n'(0)}{n(0)}[n(0)g'(n(0)) - \bar{\theta}]
\]

This expression is positive since \(n'(0) < 0, g'(n(0))n(0) = 1\) and \(\bar{\theta} \in (0, 1)\). Hence \(V_{L^C} > V_{CC}\) as \(\sigma > 0\) is the optimal choice by the home government. Therefore, the common currency area outcome is not an SPNE equilibrium.

**Proof of Proposition 6.** Given that autarchy yields private agents the lowest level of expected lifetime utility, neither government will have an incentive to impose local currency requirements. But the issue of choosing the inflation rate remains. At the steady-state equilibrium characterized in Appendix Section A.2,
the lifetime expected utility of a representative agent is given by (A.34), which is equivalent to (A.3) with the substitution of $\sigma^W$ for $\sigma$ in that expression. Hence, the proof of Proposition 5 applies so that the optimal rate of money creation is positive. Also, the steady state corresponding to $(NLC, NLC^*, 0, 0)$ is not an SPNE.

**Proof of Proposition 7.** Proposition 4 proves that if the policy configuration is $(NLC, NLC^*, 0, 0)$, then the common currency area allocation is supported. Given that this outcome yields higher expected utility than autarchy to the representative agent in either country, neither government would deviate. The common currency area allocation is a subgame-perfect Nash equilibrium insofar as the autarchic equilibrium is credible. ■

**Proof of Proposition 8.** Assume that the variance of taste shocks is zero. The fact that both countries will inflate at $\sigma = Z$ when both impose LC constraints comes directly from Proposition 2. The foreign country gets a payoff of $V^{LC^*}(Z, Z)$ in this equilibrium. To see why each country has an incentive to impose an LC constraint, consider the defection of the foreign country from $(LC, LC^*, Z, Z)$. We focus on the steady-state equilibrium in which the home currency is the only one in circulation as in the proof of Proposition 5. Given this selection, let $V^{NLC^*}(\tilde{\sigma})$ denote the payoff of the foreign country given that the home country inflates at an optimal rate $\tilde{\sigma}$. From Proposition 2, $V^{LC^*}(Z, Z) > V^{LC^*}(0, Z)$ since the foreign government has an incentive to inflate at rate $Z$. Further, $V^{NLC^*}(Z) = V^{LC^*}(0, Z)$ given that there are no gains to having a local currency if there are no taste shocks and no inflation taxes to collect.

Now we prove that the optimal rate of inflation chosen by the home country when only the home currency has value is bigger than $Z$. We then show that $V^{NLC^*}(\tilde{\sigma})$ is smaller than $V^{NLC^*}(Z)$.

The welfare of the home country when only its currency has value is equal to

$$V^{LC}(\sigma, 0) = E_\theta \left\{ \theta \ln \left( \frac{\theta n(\sigma)(\phi + \sigma)}{(1 + \sigma)\phi} \right) + (1 - \theta) \ln \left( \frac{(1 - \theta)n^*(\phi + \sigma)}{(1 + \sigma)(1 - \phi)} \right) \right\}$$

$$- g(n(\sigma))$$

where

$$\phi = \frac{\tilde{\theta}\sigma + (1 - \tilde{\theta})}{\sigma + 2(1 - \tilde{\theta})}$$

and can be rewritten as

$$V^{LC}(\sigma, 0) = E_\theta [\theta \ln \theta + (1 - \theta) \ln(1 - \theta)] + \theta \ln n + (1 - \theta) \ln n^* + \ln \left( \frac{\phi + \sigma}{1 + \sigma} \right)$$

$$- \theta \ln \phi + (1 - \theta) \ln(1 - \phi)] - g(n(\sigma))$$

23 We are very grateful to an anonymous referee for comments on our previous version of this proof.
The derivative of this expression with respect to $\sigma$ is equal to
\[
\frac{\partial V^{LC}(\sigma, 0)}{\partial \sigma} = n'(\sigma) \left[ \frac{\bar{\theta}}{n(\sigma)} - g'(n(\sigma)) \right] + \frac{1 - \phi}{(\phi + \sigma)(1 + \sigma)} + \frac{\partial \phi}{\partial \sigma} \left[ \phi(1 - \bar{\theta}) + \sigma(\phi - \bar{\theta}) \right]
\]
Evaluating this at $\sigma = Z$ and taking into account the optimal individual decision on labor, we get
\[
\frac{\partial V^{LC}(\sigma, 0)}{\partial \sigma} \bigg|_{\sigma = Z} = \frac{\bar{\theta}^2}{1 + \bar{\theta}} \left( \frac{1 + 2\bar{\theta}}{2} \right)
\]
which is always positive for $\bar{\theta} \in [0, 1]$. Hence, the optimal rate of inflation chosen by the home country $\tilde{\sigma}$ is bigger than $Z$.

The expected utility of a representative agent in the foreign country in the regime where the home country currency has value and $\sigma$ is the money creation rate is given by
\[
V^{NLC*}(\sigma) = E_\theta \{ \theta \ln \left( c_t^{*h} \right) + (1 - \theta) \ln \left( c_t^{*f} \right) \} - g(n^*)
\]
In equilibrium,
\[
c_t^{*h} = (1 - \theta^*) \frac{n(1 - \phi)}{\phi(1 + \sigma)} \quad c_t^{*f} = \theta^* \frac{1}{1 + \sigma} n^*
\]
with $n^*$ such that $1 = n^*g'(n^*)$. By direct calculation, using the characterization of $\phi$ given above, both $c_t^{*f}$ and $c_t^{*h}$ are decreasing in $\sigma$. Hence, $V^{NLC*}(\sigma)$ is a decreasing function of $\sigma$ implying that $V^{NLC*}(\tilde{\sigma}) < V^{NLC*}(Z)$. Combining all inequalities, we get $V^{NLC*}(\tilde{\sigma}) < V^{LC*}(Z, Z)$.

Thus the foreign country does not have an incentive to defect from the proposed equilibrium in which both countries impose LC constraints and inflate at rate $Z$. By continuity, if the variability of taste shocks is sufficiently small, then LC constraints will be imposed and inflation rates will be optimally set at $Z$. ■

A.2. World Economy with Two Currencies and no LC Constraints. In this section, we characterize the steady-state equilibrium of a world economy with two currencies in which (i) neither government imposes a LC constraint, (ii) home old agents receive transfers from their government, and (iii) foreign agents do not receive transfers from their government. In principle, goods in each of the countries can be purchased using either currency. Hence, period $t$ prices are given by $(p_t^h, p_t^{*h}, p_t^f, p_t^{*f})$ where $p_t^h$ is the price of home goods in home currency, $p_t^{*h}$ is the price of home goods in foreign currency, $p_t^f$ is the price of foreign goods in home currency, and $p_t^{*f}$ is the price of foreign goods in foreign currency.
The optimization problem. For the representative agent in home country, the optimization problem is

\[
\max_{c^h_{t+1}, c^f_{t+1}, \alpha, \beta, \gamma} E_0[\theta \ln c^h_{t+1} + (1 - \theta) \ln c^f_{t+1}] - g(n_t)
\]

subject to

\[
\alpha p^h_t n_t + \tau_{t+1} = \beta c^h_{t+1} p^h_{t+1} + \gamma c^f_{t+1} p^f_{t+1} \tag{A.5}
\]

\[
(1 - \alpha) p^h_t n_t = (1 - \beta) c^h_{t+1} p^h_{t+1} + (1 - \gamma) c^f_{t+1} p^f_{t+1} \tag{A.6}
\]

where \(\alpha\) is the share of the sales of his production of good \(h\) realized with home currency, \(\beta\) is the share of his consumption (when old) of good \(h\) realized with home currency, \(\gamma\) is the share of his consumption (when old) of good \(f\) realized with home currency.

The first-order conditions imply

\[
\frac{\theta}{c^h_{t+1}} = \lambda \beta p^h_{t+1} + \mu (1 - \beta) p^h_{t+1} \tag{A.7}
\]

\[
\frac{1 - \theta}{c^f_{t+1}} = \lambda \gamma p^f_{t+1} + \mu (1 - \gamma) p^f_{t+1} \tag{A.8}
\]

\[
g'(n_t) = \lambda \alpha p^h_t + \mu (1 - \alpha) p^h_t \tag{A.9}
\]

\(\lambda\) and \(\mu\) are the multipliers associated with (A.5) and (A.6). For an interior equilibrium the rate of return on holding the currencies must be equal, which implies

\[
\frac{p^h_t}{p^h_{t+1}} = \frac{p^h_{t+1}}{p^h_{t+1}} = \frac{p^f_t}{p^f_{t+1}} = \frac{p^f_{t+1}}{p^f_{t+1}} \tag{A.10}
\]

From the law of one price,

\[
\frac{e_t p^h_{t+1}}{p^h_t} = \frac{e_t p^h_{t+1}}{p^f_t} \tag{A.11}
\]

Combining these conditions, the nominal exchange rate must be constant, \(e_t = e\) for all \(t\).

Given (A.10) and (A.11), the consolidated budget constraint can be written as

\[
p^h_{t+1} c^h_{t+1} + p^f_{t+1} c^f_{t+1} = p^h_t n_t + \tau_{t+1} \tag{A.12}
\]
which, using (A.7)–(A.9), implies

\begin{align}
1 &= g'(n_t) \left[ \frac{p^h_t n_t + \tau_{t+1}}{p^h_t} \right] \\
(A.13) \\
\frac{c^h_{t+1}}{p^h_{t+1}} &= \theta \left( \frac{p^h_t n_t + \tau_{t+1}}{p^h_{t+1}} \right) \\
(A.14) \\
\frac{c^f_{t+1}}{p^f_{t+1}} &= \frac{(1 - \theta) \left( p^h_t n_t + \tau_{t+1} \right)}{p^f_{t+1}} \\
(A.15)
\end{align}

The corresponding equations for the representative foreign agent are

\begin{align}
1 &= n_{t}^* g'(n_{t}^*) \\
(A.16) \\
\frac{c^h_{t+1}^*}{p^h_{t+1}} &= \frac{(1 - \theta) p^f_t n_{t}^*}{p^h_{t+1}} \\
(A.17) \\
\frac{c^f_{t+1}^*}{p^f_{t+1}} &= \frac{\theta p^f_t n_{t}^*}{p^f_{t+1}} \\
(A.18)
\end{align}

*Market clearing.* The money market conditions for the goods and the money markets are

\begin{align}
n_{t} &= c^h_{t} + c^h_{t}^* \\
(A.19) \\
n^*_t &= c^f_{t} + c^f_{t}^* \\
(A.20) \\
M_t &= \alpha p^h_t n_{t} + (1 - \alpha^*) \ p^f_t n^*_t \\
(A.21) \\
M^*_t &= (1 - \alpha^*) p^h_t n_{t} + \alpha p^f_t n^*_t \\
(A.21) \\
M^W_t &= M_t + M^*_t = p^h_t n_{t} + p^f_t n^*_t \\
(A.22)
\end{align}

where \( M_t, M^*_t, \text{ and } M^W_t \) refer to the home, foreign, and world money supplies, respectively, all evaluated in home currency.

*Steady-state equilibrium.* We characterize the steady state of this economy \((n, n^*, c^h, c^f, c^h^*, c^f^*)\) and the following transfer policy followed by the home government,

\begin{align}
\tau_{t+1} &= \sigma^W M^W_t \\
(A.23)
\end{align}

That is, we have specified government policy in terms of choosing the growth rate of the world money supply. This is for analytic convenience.
To find an equilibrium, we conjecture that home agents receive a fixed fraction \( \phi \) of the world money supply. From market clearing, we find

\[
\phi = \frac{\bar{\theta} \sigma^W + (1 - \bar{\theta})}{2(1 - \bar{\theta}) + \sigma^W}
\]

This allows us to write lifetime expected utility as

\[
V^{KW}(\sigma^W, 0) = E_0 \left\{ \theta \ln \left( \frac{\theta n(\sigma^W)(\phi + \sigma^W)}{(1 + \sigma^W)\phi} \right) + (1 - \theta) \ln \left( \frac{(1 - \theta)n*(\phi + \sigma^W)}{(1 + \sigma^W)(1 - \phi)} \right) \right\} - g(n(\sigma^W))
\]

In deriving this expression, we have used the fact that in equilibrium all prices will grow at the growth rate of the world money supply.

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