Exchange rate regime credibility, the agency cost of capital and devaluation

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Abstract

Hong Kong pegs their currency to the US dollar with a currency board. In the wake of the Asian financial crisis in 1997, the US–Hong Kong interest rate differential jumped from 1 2% to 4–6%. Investors feared Hong Kong would abandon the peg.

This paper analyzes the crucial role of credibility in a stochastic dynamic rational expectations regime switch model. I parameterized the model using estimates of the exchange rate process for Hong Kong. The model generated interest rate differentials are consistent with the interest differentials in Hong Kong before the Asian financial crisis in July of 1997 but not after the crisis. © 2002 Elsevier Science B.V. All rights reserved.

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0. Introduction

Joseph Yam, Chief Executive Hong Kong Monetary Authority in the Wall Street Journal 8/20/98

… the actions of currency speculators can be blamed for a significant part of the interest-rate premium in the Hong Kong dollar over the US dollar. This premium has been unfairly attributed to the possibility that the government, operating under a currency-board system, may lose its nerve…

Agency problems increase the cost of capital when the agent cannot credibly commit to the principals that he would not take actions that harm them. When the Central Bank does not credibly commit to maintaining an exchange regime, investors demand

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an agency currency premium in addition to the normal cost of capital. This paper
analyzes the crucial role of imperfect credibility in a currency crisis with a stochastic
dynamic rational expectations regime switch model. The innovation in the paper is to
isolate and quantify the cost of imperfect credibility. If the country would credibly
commit, e.g., by dollarizing, then the time-series average of the (risk-adjusted) cost
of domestic capital would equal the time-series average cost of capital in the foreign
country. The failure to commit adds an agency currency premium to the country’s cost
of capital. Numerical solution exercises indicate that the agency currency premium is
small. It increases the average cost of capital in Hong Kong by approximately 1.2%.
This is consistent with the average interest rate differential in Hong Kong before the
Asian financial crisis in July 1997. But the model-generated agency currency premium
is not large enough to explain the observed interest rate differentials of 4–6% after July
1997. The paper’s main empirical result—that a lack of credible commitment cannot
generate large interest rate differentials in a sound currency regime—is robust.

The model in this paper is stylized, but the results are rich. It generates multiple
rational expectations equilibria and a variety of patterns linking the exchange rate to the
interest rate differential. Investors and the Bank optimize. Investors fear devaluation and
demand a currency premium because the Bank cannot make a credible commitment.
The Bank abandons the exchange regime when the expected present value of the
deadweight welfare loss from the agency currency premium outweighs the expected
present value of the benefit from remaining in the regime. The agency premium and
the Bank’s abandonment threshold are endogenous. The rational expectations equilibria
function is S shaped; a fully credible equilibrium always exists, and usually less than
full credibility equilibria also exist. In a less than full credibility equilibrium, the agency
cost is a simple monotonically increasing function of the deviation of the exchange rate
from central parity. The interest rate differential is a non-monotonic function of the
deviation of the exchange rate from central parity that depends on the agency premium
and the Bank’s control rule.

The basic technical difference between this model and most models with an optimiz-
ing policy maker is that this model has richer dynamics. The model has no analytic
solution. I compute and analyze the solution with numerical techniques. I use Tauchen’s
(1986) algorithm to approximate the continuous state Markov regime switch process
for the exchange rate with a finite state Markov regime switch process. I modify Nys-
trom’s method for solving integral equations to numerically approximate the solution
to the Bank’s stopping problem. Then I search the finite state space for all the rational
expectations equilibria.

The paper is organized as follows: Section 1 presents the model and defines equi-
librium. Section 2 gives the results from numerical solutions. Section 3 gives the con-
clusion. Appendix A gives the details of the solution algorithm. Appendix B presents
sensitivity analysis results to the model parameterization.

1. Model and equilibrium

This is a simple market equilibrium model of the exchange rate. The exchange
rate follows a Markov regime switch process. In the “managed regime”, the Bank
follows a feedback rule that forces the exchange rate toward central parity. In the floating regime, the exchange rate follows a random walk. The decision to abandon the regime (by assumption) is irreversible, so the switch occurs only once. Investors form expectations of future values of the exchange rate. Their expectations depend on the likelihood the bank will abandon the regime. The Central Bank’s decision to abandon depends on the investors’ expectations.

1.1. Specification

**Exchange rate.** Define the exchange rate, $X$, as units of domestic currency per unit of the foreign currency. Let $X_0$ denote the central parity, and $X^*$ the Bank’s abandonment threshold. And let the lower case letters denote the natural logarithms, i.e., $x^* \equiv \ln(X)$, $x_0 \equiv \ln(X_0)$, and $x^* \equiv \ln(X^*)$.

**Returns.** The realized (logarithmic) return on an $n$-period default-free discount bond in the foreign country is

$$ i(n)_t^* = \ln(1/B^*(n)_t), $$

where $B^*(n)_t$ is the bond price.

The realized (logarithmic) return on an $n$-period local investment denominated in the foreign currency is

$$ r_{t+n} - \Delta(n)x_{t+n} = \ln \left( \frac{P_{t+n}X_t}{P_tX_{t+n}} \right), $$

the return, $r_{t+n}$, minus the (logarithmic) change in the exchange rate. Here $P_t$ denotes the price of the investment and $P_{t+n}$ the total payoff.\footnote{For example, if the investment is an equity investment the total payoff includes reinvested dividends.}

**Stochastic process for the exchange rate.** In any period the exchange rate is either in the managed regime, in the transition to a floating regime, or in the floating regime. The switch point—the Bank’s threshold—and investors’ expectations are endogenous. Section 1.2 presents investor and Bank behavior and Section 1.3 defines a rational expectations equilibrium.

Define $z \equiv \ln(Z)$ as a mean zero independently and identically distributed shock to the exchange rate.

**Managed regime.** In the managed regime, the exchange rate follows the mean-reverting process,\footnote{I model the managed regime as a mean-reverting process. This a linear approximation to “fixed” or “target zone” regimes. I parameterize the feedback rule with data from the “fixed” rate regime in Hong Kong. Hong Kong’s exchange rate, which is controlled by a Currency Board, varies, see Section 2. Alternatively, the mean-reverting process can be interpreted as an approximation to a target zone regime. Garber and Svensson (1994) argue that the feedback rule is a good empirical approximation because most intervention in a target regime is intramarginal.}

\[
x_{t+1} - x_0 = (1 - a)(\{x_t - x_0\} + z_{t+1}); \quad \begin{cases} x_t \leq x^*, \\ (1 - a)(\{x_t - x_0\} + z_{t+1}) \leq x^* - x_0. \end{cases}
\]
If the country is in the regime at the beginning of the period, \( x_t \leq x^* \), and the current shock does not drive the exchange rate out of the region, \((1-a)(\{x_t - x_0\} + z_{t+1}) \leq x^* - x_0\), then the Bank follows the feedback rule, \(^3\)

\[
\Delta x_{t+1} = -a(\{x_t - x_0\} + z_{t+1}) + z_{t+1}; \quad a \in (0, 1),
\]

that forces the exchange rate toward central parity.

**Abandonment.** The currency always depreciates when the Bank abandons the regime. If the current shock, \( z^* \), drives the exchange rate over the threshold, \((1-a)(x_t - x_0 + z^+_{t+1}) > x^* - x_0\), then the Bank abandons the regime \((a = 0)\). In the basic specification, there is no fundamental disequilibrium. In the extended version of the model, the currency is overvalued and it depreciates by \(d\%\) \(^4\) plus the shock,

\[
x^+_{t+1} = x_t + d + z^+_{t+1}; \quad \left\{ \begin{array}{l} x_t \leq x^*, \\ (1-a)(\{x_t - x_0\} + z^+_{t+1}) > x^* - x_t \end{array} \right. 
\]

to reach equilibrium in the transition period.

The shock that triggers abandonment, \( z^* \),

\[
x^+_{t+1} - \{x_t + d\} = z^+_{t+1} > z^+_{t+1} - a(\{x_t - x_0\} + z^+_{t+1}) > x^* - x_t > 0,
\]

is positive so the currency depreciates in the transition period even if it is not overvalued, i.e., \(d = 0\).

**Floating regime.** After the Bank abandons the target regime, the (log of the) exchange follows a random walk,

\[
x_{t+1} = x_t + z_{t+1}; \quad x_{t-j} > x^*.
\]

**Notation.** To simplify the notation define \( v \) as the exchange rate process in the managed fixed rate regime and \( w \) as the exchange rate process in the floating regime,

\[
v_{t+1} = (1-a)(v_t + z_{t+1}), \quad w_{t+1} = w_t + z_{t+1}
\]

and define \( y \) and \( y^* \)

\[
y_t \equiv (x_t - x_0), \quad y^* \equiv (x^* - x_0)
\]

as the (logarithmic) deviation of the exchange rate and the threshold from central parity.

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\(^3\) The unconventional feedback rule allows the Bank to offset part of the current shock, \( \Delta x_{t+1} = -a(\{x_t - x_0\} + z_{t+1}) + z_{t+1} \). In a discrete time model, the distinction is quantitatively important. It makes the conditional variance of the exchange rate smaller in the exchange regime (which fits stylized facts.) The qualitative results are similar if I use the standard feedback rule, \( \Delta x_{t+1} = -a(x_t - x_0) + z_{t+1} \).

\(^4\) The exogenous currency depreciation from fundamental overvaluation can be considered as an iid random variable with no change in the model.
Using these definitions the stochastic process for the (log of the) exchange rate can be compactly written as,

\[
y_{t+1} = \begin{cases} 
  v_{t+1}; & v_{t+1} \leq y^*, v_t \leq y^* \text{ exchange regime,} \\
  w_{t+1} + d; & v_{t+1} > y^*, v_t \leq y^* \text{ transition,} \\
  w_{t+1}; & v_t > y^* \text{ floating.}
\end{cases}
\]

1.2. Behavior

Assumptions.

(1) Agents are risk neutral.\(^6\)

(2) The Bank’s decision to abandon the managed regime is irreversible.

(3) Market Failures
  (a) The Central Bank cannot credibly commit to maintaining the regime, or
  (b) The Central Bank cannot credibly commit to maintaining the regime and the
      currency is overvalued by \(d\).

1.2.1. Investors’ problem

The investors’ problem is to choose the investment with the highest expected return. The expected \(n\)-period return on any domestic investment,

\[ E(r_{t+n}) \geq i^*(n) + E_t \Delta x_{t+n} \]

must equal or exceed the \(n\)-period risk-free return on the foreign bond plus the expected change in the exchange rate.

Exchange rate expectations. Investors’ expectations of the exchange rate are forecasts from the Markov regime switch process. Their expectations depend on their perception of the Bank’s behavior.

One-period forecasts. If the Bank is in the regime at the beginning of the period, then it either continues in the regime, or it stops. The expectation of the exchange rate next period conditional on being in the fixed regime this period equals,\(^7\)

\[
E(y_{t+1} | v_t = y_t \leq y^*) = E(v_{t+1} | v_{t+1}, v_t \leq y^*) \text{ prob}(v_{t+1} \leq y^* | v_t \leq y^*) \hspace{.5cm} \text{(continue)} \\
+ E(w_{t+1} + d | v_{t+1} > y^*, v_t \leq y^*) \text{ prob}(v_{t+1} > y^* | v_t \leq y^*) \hspace{.5cm} \text{(stop)}
\]

the expectation of the exchange rate in the continuation region plus the expectation of the exchange rate in the stopping region.

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5 The initial state in the random walk process is \(v_t\).

6 Alternatively, I could assume markets span the payoff space and expectations are under the “Martingale equivalent” probability measure so that “risk adjusted” returns are equalized.

7 Appendix A shows the exchange rate forecasts from a finite state Markov chain representation.
**J-period forecasts.** At the beginning of period \( t + j \), the Bank is either in the fixed regime, or it has stopped. If it is in the fixed regime, then it either continues or it stops. If it stops, then it remains in the stopping regime. The expectation of the exchange rate \( j \) periods in the future is

\[
E(y_{t+j}|v_t = w_t = y_t \leq y^*)
\]

\[
= E(v_{t+j}|v_{t+j}, v_{t+j-1}, \ldots, v_t \leq y^*) \ \text{prob}(v_{t+j}, v_{t+j-1}, \ldots, \{\text{continue in } j\})
\]

\[
+ E(w_{t+j} + d|v_{t+j} > y^*, v_{t+j-1}, \ldots, v_{t+1} \leq y^*) \ \text{prob}(v_{t+j} > y^*, v_{t+j-1} \leq y^*, \ldots, v_{t+1} \leq y^*) \ \{\text{stop in } j\}
\]

\[
+ E(w_{t+j-1} + d|v_{t+j-1} > y^*, v_{t+j-2}, \ldots, v_t \leq y^*) \ \text{prob}(v_{t+j-1} > y^*, v_{t+j-2} \leq y^*, \ldots, v_t \leq y^*) \ \{\text{stopped in } j - 1\}
\]

\[
\vdots
\]

\[
+ E(w_{t+1} + d|v_{t+1} > y^*, v_t \leq y^*) \ \text{prob}(v_{t+1} > y^*, v_{t+1} \leq y^*) \ \{\text{stopped in 1}\}
\]

As the forecasting horizon gets long (\( j \) goes to infinity) the probability of staying in the continuation region gets small (converges to zero) and the forecast of the exchange rate goes to a constant.

**Equilibrium expected returns.** In equilibrium, the expected \( j \)-period return on any domestic investment minus the \( j \)-period risk-free return on the foreign bond,

\[
E_t r_{t+j} - i(j)^* = E_t \Delta(j)x_{t+j} = E_t(y_{t+j} - y_t) \quad \forall j
\]

equals the expected change in the exchange rate.

**Agency currency premium.** If the Bank abandons the regime, then the currency depreciates. Investors charge a premium because the Bank cannot credibly commit to maintaining the exchange regime. The premium fairly prices the asset, but it increases the country’s cost of capital. It the Bank could credibly commit, there would be no agency premium.

Assume the currency is not overvalued, i.e., \( d = 0 \). Define the \( j \)-period agency currency premium as

\[
a(j, y_t) = E_t[y_{t+j}|y_t \leq y^*, d = 0] - E_t[v_{t+j}|y_t],
\]

the expected exchange rate under an imperfect credibility regime, \( E_t y_{t+j} \), minus the expected exchange rate under a perfect credibility regime, \( E_t v_{t+j} \). The agency premium is always positive and increasing (non-decreasing) in maturity (\( j \)) as long as the Bank’s threshold is greater than the central parity, \( y^* > 0 \).\(^8\)

\(^8\) Ozkan and Sutherland use a symmetric loss function. In their model, shocks that drive the interest rate too high or too low lead to abandonment. I use an asymmetric loss function. Only shocks that depreciate the currency cause the Bank to abandon the exchange regime.
One-period premium. The one-period agency premium equals the expected value of the exchange rate in the upper tail of the distribution, \( y > y^* \), under the random walk regime minus the expected value of the exchange rate under the feedback rule.

\[
a(1, y_t) = \left[ E(v_{t+1}|v_{t+1}, v_t \leq y^*) - E(v_{t+1}|v_{t+1}, v_t \leq y^*) \right] \\
\quad \text{prob}(v_{t+1} \leq y^*|v_t \leq y^*) \\
+ \left[ E(w_{t+1}|v_{t+1} > y^*, v_t \leq y^*) - E(v_{t+1}|v_{t+1} > y^*, v_t \leq y^*) \right] \\
\quad \text{prob}(v_{t+1} > y^*|v_t \leq y^*) \\
= \left[ E(w_{t+1}|v_{t+1} > y^*, v_t \leq y^*) - E(v_{t+1}|v_{t+1} > y^*, v_t \leq y^*) \right] \\
\quad \text{prob}(v_{t+1} > y^*|v_t \leq y^*) > 0,
\]

since \( w_{t+1} = v_{t+1}/(1 - a) \).

If a shock drives the exchange over the threshold, then the Bank abandons its feedback policy and the currency depreciates more than it would have if the bank maintained the feedback policy. Investors demand a premium to compensate for the additional depreciation. If the Bank could assure investors that they would never abandon the regime, then investors would not demand a premium. The premium is an agency cost.

\( J \)-period agency premium. The agency premium for any period \( j \),

\[
a(j, y_t) = \left[ E(w_{t+j}|v_{t+j} > y^*) - E(v_{t+j}|v_t) \right] \\
\quad \text{prob}(v_{t+j} > y^*, v_{t+j-1}, \ldots, v_{t+1} \leq y^*|v_t) \\
+ \left[ E(w_{t+j-1}|v_{t+j-1} > y^*) - E(v_{t+j-1}|v_t) \right] \\
\quad \text{prob}(v_{t+j-1} > y^*, v_{t+j-2}, \ldots, v_{t+1} \leq y^*|v_t) \\
\cdots \\
+ \left[ E(w_{t+1}|v_{t+1} > y^*) - E(v_{t+1}|v_t) \right] \text{prob}(v_{t+1} > y^*|v_t)
\]
equals the sum of the one-period agency premiums.

Agency cost of capital. The agency currency premium increases the cost of capital on all domestic investment. The expected \( j \)-period return on a domestic investment can be decomposed into

\[
E_{tr_{t+j}} = i_{t+j} + E(v_{t+j} - v_t|v_t \leq y^*) + a(j, y_t) + d(j, y_t),
\]
the foreign risk-free return plus the expected change in the exchange rate under a credible fixed exchange regime plus the agency currency premium plus the expected depreciation from fundamental currency disequilibrium, \( d(j, y) \). Notice that the agency currency premium and the expected depreciation from disequilibrium are always positive, but the interest rate differential can be negative.
1.2.2. The Central Bank’s problem

The Bank’s problem is to choose the abandonment threshold, \( y^* \), given investors’ expectations.

**Benefit.** The country receives a flow welfare benefit, \( B \), from remaining in the exchange regime. The benefit may come from an observable signal that the country relinquished discretionary monetary policy, or it may come from foreign trade arrangements. Whatever the reason, if there were no benefits to the exchange regime, then there would be no reason to remain in the exchange regime.

**Cost.** The country pays a higher cost of capital because the Bank cannot credibly commit to maintaining the target zone regime. Define the welfare loss from the agency cost of capital as

\[
A(y_t) \equiv \sum_j q_j a(j, y_t),
\]

a weighted average of the agency currency premia. The weights, \( q_j \), reflect the maturity structure of the capital.

**Utility.** The flow utility to the Bank is

\[
U(B, A(y_t)) = B - kA(y_t); \quad k \in (0, 1),
\]

the benefit, \( B \), minus the agency cost of capital, \( A(y) \). The utility measures the “welfare” gain to the country. Notice this is a standard agency cost problem. The country receives a benefit from the exchange regime and wants to maintain the regime. If it could credibly commit, then investors would not charge an agency premium and the country would never abandon.

**The stopping problem.** The irreversibility of the decision to abandon the exchange regime makes the Bank’s problem asymmetric and dynamic. The Bank could abandon as soon as the flow utility goes negative. But if the Bank abandons today, it loses the potential to enjoy future positive utility from remaining in the regime. The situation might improve tomorrow, and if it does not, then the Bank always has the option to abandon tomorrow.

The Bank solves the dynamic stopping problem, \(^{10}\)

\[
f(y_t) = \max[0, B - kA(y_t) + \beta \mathbb{E} f(y_{t+1} | y_t)]; \quad 0 < \beta < 1,
\]

where \( f(y) \geq 0 \) is the option value of waiting. At the abandonment threshold, \( y^* \), the expected present value of the benefit from remaining in the regime equals the (weighted) expected present value of the deadweight welfare loss from the agency cost,

\[
B/(1 - \beta) = k \mathbb{E} \left[ \sum_{j=0}^{\infty} \beta^j A(y_{t+j} | y_t^*) \right].
\]

\(^9\) A more complicated specification would make the benefit a function of the state variable. I chose the simplest specification and focused on the agency cost. Ozkan and Sutherland also use the simple constant flow benefit specification.

\(^{10}\) The Bank in Ozkan and Sutherland (1995, 1998) papers also solves a dynamic stopping problem.
At the threshold, the option value of waiting is zero, \( f(y^*) = 0 \). If the shock drives the controlled value of \( y \) over the threshold, \( y^* \),

\[ y^* < v_{t+1} = (1 - a)(y_t + z_t + 1), \]

then the expected present value of the costs exceeds the benefits and the Bank abandons the exchange regime.

### 1.3. Rational expectations equilibrium

A rational expectations equilibrium is a pair:

1. a set of beliefs, \( G(y_t, y^*) \), so that investors’ exchange rate expectations are correct, and

2. a threshold, \( y^*(y_t, A(G(y_t, y^*))) \), that solves the Bank’s problem given investors’ beliefs.

### 2. Numerical solutions

No closed-form solution exists to evaluate investors’ expectations or to solve the Bank’s stopping problem. And, of course, no closed-form solution exists for the rational expectations equilibrium. I numerically solve for the rational expectations equilibria.

I approximate the continuous state space for the exchange rate with a discrete state space and solve the investors’ and the central bank’s problems. I search the discrete state space for the rational expectations equilibria. Appendix A gives the details.

Three functions summarize the results: the rational expectations equilibria function, the agency currency premium, and the interest rate differential function. The rational expectations equilibria function has an S shape. A fully credible equilibrium always exists. If investors believe that the Bank will never abandon the exchange regime, then they charge no agency premium and the Bank never abandons the regime. The model parameterized with estimates from Hong Kong also generates two less than fully credible rational expectations equilibria—a “high” confidence equilibrium and a “low” confidence equilibrium. The S-shaped equilibrium function is robust with respect to the parameterization. The number and location of the imperfect credibility equilibria are sensitive to the parameterization, see Appendix B.

The agency currency premium and interest differential functions trace out the agency premium or interest differential as a function of the exchange rate in the two less than fully credible rational expectations equilibria. The agency currency premium is a simple monotonically increasing function of the exchange rate—as the exchange rate depreciates the probability that a shock will drive the exchange rate over the threshold increases so the agency premium increases. A main result in the paper is that the agency currency premium is small. It is not large enough to explain large interest rate differentials. This result is robust with respect to the model parameterization, see Appendix B. The interest differential is a complicated function of the exchange rate—it depends on the agency cost, the Bank’s feedback rule, and the fundamental currency disequilibrium—there is no simple pattern.
Fig. 1. Exchange rates and interest differentials for Hong Kong.

Data from Hong Kong. The Hong Kong monetary authority is a “currency board”. They have sufficient reserves to defend the exchange regime if they choose to. Fig. 1 shows the interest rate differential and exchange rate from Hong Kong.

The top panel shows a time-series plot of the (log of the) exchange rate (price of a US$ in Hong Kong $s) and the 3-month interest rate differential (Hong Kong rate – US rate) from 1986:2 to 1998:4. Notice that the monetary authority allowed more variability in the exchange rate (dotted line) in the 86–92 period and they allowed the currency to appreciate by about 1% relative to the US$. The exchange rate was relatively stable (but not constant) from 92:2 through the end of the sample in 98:4. The interest rate differential jumps from about \( \frac{1}{2} \)% to 5% after Thailand abandoned its exchange regime in July of 1997. The bottom panels show scatter diagrams of the interest rate differential and the (log of the) exchange rate for the subsamples

\[ 11 \] The data are from DATASTREAM.
\[ 12 \] This is the longest sample I could find in DATASTREAM.
Table 1
OLS estimates of simple feedback rules

\[ \Delta x_{t+1} = a_0 - ax_t + e_{t+1} \]
\[ a_0 = ax_0 \]

<table>
<thead>
<tr>
<th>Sample</th>
<th>( a_0 ) (P value)</th>
<th>(-a) (P value)</th>
<th>( \geq x_0 )</th>
<th>SER</th>
</tr>
</thead>
<tbody>
<tr>
<td>86:2–98:4</td>
<td>0.343 (0.031)</td>
<td>0.167 (0.030)</td>
<td>2.049</td>
<td>0.0022</td>
</tr>
<tr>
<td>92:2–98:4</td>
<td>1.647 (0.000)</td>
<td>0.805 (0.000)</td>
<td>2.046</td>
<td>0.0010</td>
</tr>
</tbody>
</table>

1992:2–1997:2 and 1992:2 through the end of the sample in 1998:4. Before the Asian crisis in July of 1992 (the left-hand panel) the interest rate differential averaged around 1.25%—no observations exceeded 1.25%, and only one observation was > 1.0%. After the crisis, no observation was < 1%, and two-thirds (four) of the observations exceeded 2.0%.

Table 1 shows the OLS estimates of simple feedback rules of the form, for the full sample and the subsample after 1992. For the subsample after 1992 the feedback coefficient, 0.8, is fairly large and the standard error of the regression (SER) is small. Hong Kong followed an aggressive exchange policy that kept the rate close to central parity after July of 1992. The implied central parity value, \( x_0 \), equals the subsample mean. For the full sample the feedback coefficient, \( a \), is small and the standard error of the regression is larger. The implied central parity value is slightly above the mean of the full sample.

**Model parameterization**

**Exchange rate process.** I used the estimates of the feedback rule after 1992:2 to parameterize the exchange rate process,

\[ \Delta x_{t+1} = -a(x_t - x_0) + (1 - a)z_{t+1}, \]
\[ a = 0.805, \]
\[ \sigma_z = 0.053 = \sigma_e/(1 - a) = 0.001/(1 - 0.805), \]
\[ z \sim t(0, \sigma, 5). \]

I chose a “fat-tailed” t distribution with 5 degrees of freedom for the error process.

Fig. 2 shows the discrete Markov chain approximation to the conditional probability distributions for the exchange rate process centered at parity, i.e., \( x_t = x_0 \Rightarrow y_t = 0 \).

The mean-reverting process has most of its mass concentrated close to central parity. The active feedback policy substantially reduces the conditional volatility. The conditional standard deviation of the mean-reverting process is only 20% of the conditional standard deviation of the random walk process. If the Bank abandons the exchange regime, then the probability of a large change in the exchange rate is much higher.

\[^{13}\text{Tauchen (1986) shows that a discrete state space of 10 provides a good approximation for a univariate AR}(1)\text{ process. I use 51 states because I want a good approximation to the continuous state stopping problem.}\]
The central bank

Utility function

\[ U(R, A(y_t)) = B - 0.4A(y_t), \]
\[ B = (1.15)^{1/4} - 1, \]
\[ A(y_t) = \sum_{j=1}^{20} \frac{1}{j} a(j). \]

I set the reputation benefit to 15% a year and I used a 5-year horizon with arithmetically declining weights for the agency cost function. I set the trade-off parameter between benefit and agency cost at 0.4.

Stopping problem. The bank abandons the exchange regime when the value of the option to wait, \( f(y) \), is zero,

\[ f(y_t) = \max[0, B - kA(y_t) + \beta Ef(y_{t+1}|y_t)]; \]

\[ \beta = 0.90^{1/4}. \]

The time discount rate, \( \beta^{-1} - 1 = 0.11 \), is slightly less than the flow reputation effect.

Appendix B shows the solution for perturbations of the parameters.

Rational expectations equilibria. The model generates multiple equilibria. Fig. 3 plots the option value of waiting, \( f(y) \), and the conditional probability density of the exchange rate, \( y \), centered at parity. I find the equilibria by searching a discrete approximation to the state space, \( y \in \{y_1, y_2, \ldots, y_N\} \), here \( N = 51 \). I assume \( y_{N-1} = y^* \) is the threshold, and calculate investors expectations and the agency cost of capital,
$A(y|y_{N-1} = y^*)$. Then I solve the Bank’s stopping problem given the agency cost. If the option value of waiting is zero, $f(y_{N-1}) = 0$, then the investors’ expectations are correct, $y_{N-1} = y^*$, and it is a rational expectations equilibrium. If the option value of waiting is not equal to zero, then investors’ expectations are inconsistent with the Bank’s behavior and it is not a rational expectations equilibrium. Next I assume $y_{N-2} = y^*$ is the threshold and repeat the process. I search all the values of the exchange rate above parity (positive values of $y$) for rational expectations equilibria. Appendix A has the details for the solution algorithm.

Fig. 3 shows the two interior rational expectations equilibria. A high confidence equilibrium exists at $y^* = 0.44\%$. In the high confidence equilibrium, investors believe that the Bank will not abandon the target regime unless a shock causes the currency to depreciate $> 0.44\%$ from parity. Depreciation of less than half of 1% is not much. But after 1992 the Bank followed a very aggressive feedback policy so in relative terms 0.4% is a large deviation. No realizations in scatter diagrams in Fig. 1 are farther than 0.25% from central parity. From central parity, the conditional probability that a shock will exceed the high confidence threshold is very small, $\text{prob}(y_{t+1} > y^* = 0.44\%|y_t = 0) = 0.36\%$. There is also a low confidence equilibrium at $y^* = 0.17\%$.

---

14 Since I approximate the continuous state space with a discrete state space the option value at the threshold is not precisely zero, see Appendix A.

15 $\text{LNX}_X$ is log(exchange rate/sample mean) for the period 1992:2–1998:4.
The conditional probability that a shock from parity would exceed the low confidence threshold is 7.2%.

According to the model, Hong Kong was in the high confidence equilibrium after 1992. The average interest rate differential was positive—which rules out the full confidence equilibrium. And three realizations of the post-1992 Hong Kong sample violate the low confidence threshold of 0.17%—which rules out the low confidence equilibrium. No realizations violate the high confidence threshold.

**Properties of the equilibria**

*Agency premium.* The model-generated agency cost of capital is small, but nevertheless costly to the country. Fig. 4 plots the percentage agency currency premium at annual rates for a quarter and a 5-year investment at the high confidence equilibrium and the low confidence equilibrium as a function of the current exchange rate. It “averages” 16 almost 1/2%.

The agency premium depends on three factors: the maturity of the investment, the distance of current realization of the exchange rate from the threshold, and the level of the threshold. The closer the current realization of the exchange rate is to the threshold, the higher the probability of abandonment—and the higher the agency premium. Similarly, the higher the threshold the greater the loss from abandonment relative to the full credibility regime which eventually drives the exchange rate back to parity. Finally, longer maturity investments average the potential loss over the life of the investment which makes annualized agency premium less sensitive to the current value of the exchange rate.

16 Fig. 4 plots the agency premium conditional on the current state. The “average” is a descriptive summary statistic that I computed by weighting the conditional agency premium with the ergodic probabilities for the mean-reverting process. This is not the unconditional mean of the process with absorbing states.
In the high confidence equilibrium, the agency premium on the one-quarter investment reaches a little over 1% near the threshold and is only 0.45% at parity. The agency premium on the 5-year investment is not very sensitive to the current value of the exchange rate—it is \( \approx 0.45\% \). In the low confidence equilibrium, the agency costs are slightly lower even though the probability of abandonment is higher. The reason is that the expected depreciation from parity conditional on abandonment is smaller, the lower the threshold is.

**Interest rate differentials.** The interest differential equals the expected change in the exchange rate—which can be decomposed into (1) the expected change under a full credibility regime plus (2) the agency currency premium plus (3) any expected depreciation from a fundamentally overvalued currency. The full credibility expected change in the exchange rate is the forecast generated by the Bank’s feedback rule, \( E\Delta y = -0.8y \), which leads to a negative correlation between interest differentials and the exchange rate. When the exchange rate is above parity, the Bank forces it back to parity which makes the currency appreciate and when it is below parity, the Bank forces it to parity which causes the currency to depreciate. The agency premium is a positive increasing function of the exchange rate—the closer to the threshold, the higher is the agency premium. Expected depreciation from fundamental overvaluation is also a positive increasing function of the exchange rate.

The agency premium and fundamental overvaluation are unobservable. Interest rate differentials are observable. Fig. 5 shows the model-generated 3-month interest rate differential function (at the high confidence equilibrium) superimposed on the interest rate differential scatter diagram in Fig. 1. The negatively sloped line labeled \( d = 0.0\% \) at the bottom of the picture is the model-generated interest differential function when expected depreciation from fundamental currency overvaluation is zero. Agency premiums are small and the Bank’s feedback rule dominates the interest differential function. The model-generated interest differentials are roughly consistent with the small observed interest rate differentials in the scatter diagram prior to the Asian crisis in 1997:2.

The positively sloped line labeled \( d = 3.5\% \) at the top of the picture is the model-generated interest differential function when expected depreciation from fundamental currency overvaluation is 3.5%. The expected loss from depreciation when the currency is overvalued dominates the interest differential function. The function is consistent with the scatter plot (of course there are only six observations) of high interest rate differentials after the Asian crisis in July of 1997.

3. **Summary and conclusions**

This paper analyzes the crucial role of imperfect credibility in a currency crisis with a stochastic dynamic rational expectations regime switch model. The paper has two innovations: (1) It specifies the cost of imperfect commitment, and (2) it quantifies the cost of imperfect commitment. A main empirical result is that the agency currency
premium is small if the only failure is the inability to commit. It is too small to explain large interest rate differentials.

The model-generated data are consistent with the interest differentials in Hong Kong before the Asian financial crisis in July of 1997. After July 1997 the model-generated agency currency premiums are too small to explain the observed interest rate differentials. Adding exogenous fundamental currency disequilibrium of 3.5% to the model is enough to generate larger interest rate differentials that are consistent with the observed interest rate differentials from Hong Kong after July of 1997.

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Appendix A. Solving for a rational expectations equilibrium

I divide the continuous state space into a discrete state space and numerically solve for the rational expectations equilibria. Multiple rational expectations equilibria exist. I start by assuming a threshold and solve the investors’ problem. The investors’ ex-
pectations (conditional on the assumed threshold) imply the currency agency premium. Then, I solve the Bank’s problem for the threshold given the agency premium. If the threshold solution to the Bank’s problem matches the assumed threshold in the investors’ problem, then it is a rational expectations equilibrium.

**Step 1:** Approximate the continuous process of the state variable with a finite state Markov chain. Using the notation of Section 1.1, let \( v \) denote the exchange rate process in the target zone regime and \( w \) the exchange rate process in the floating regime,

\[
\begin{align*}
v_{t+1} &= (1 - a)(v_t + z_{t+1}), \\
w_{t+1} &= w_t + z_{t+1},
\end{align*}
\]

where, \( z \), is a mean-zero iid shock. And define \( y \) and \( y^* \),

\[
\begin{align*}
y_t &\equiv (x_t - x_0), \\
y^* &\equiv (x^* - x_0)
\end{align*}
\]

as the (logarithmic) deviation of the exchange rate and the threshold from central parity.

Approximate the continuous support, \( y \), with a discrete support. Divide the interior into \( N - 2 \) equally spaced intervals. The interior intervals plus the two tails make up the \( N \) states. Now compute the Markov transition probabilities for the target zone regime and the random walk regime so that the Markov chains approximate the continuous conditional distributions. I use Tauchen’s (1986) algorithm. Let

\[
\begin{align*}
p_{ij} &\in P \equiv \text{Prob}(v_{t+1} \in j | v_t \in i) \\
q_{ij} &\in Q \equiv \text{Prob}(w_{t+1} \in j | w_t \in i)
\end{align*}
\]

denote the transition probability of moving from state \( i \) to state \( j \) for the mean reverting process. And let,

\[
\begin{align*}
Y &\equiv \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}, \\
V &\equiv \begin{bmatrix} v_1 \\ \vdots \\ v_N \end{bmatrix}, \\
W &\equiv \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}
\end{align*}
\]

denote the values of the state variable. Notice the value of the states are same for both processes. The probabilities are different.

**Step 2:** The investors’ problem. The investors’ problem is to evaluate the expected change in the exchange rate given a conjecture about the Bank’s threshold.
Define, $y^* \equiv v_j*$ as the conjectured threshold. And partition $Y$ into states in the continuation region, $y_j \leq y^* \in Y_c$ and states in the transition region, $y_j > y^* \in Y_a$.

$$Y = \begin{bmatrix} Y_c \\ Y_a \end{bmatrix} = \begin{bmatrix} v_1 \\ \vdots \\ v_j* = y^* \\ w_{j*+1} + d \\ \vdots \\ w_N + d \end{bmatrix}$$

And make a conformable partition for the transition probabilities,

$$T = \begin{bmatrix} P_{cc} & Q_{ca} \\ 0 & I_{aa} \end{bmatrix}.$$  

The first $c$ rows of the transition matrix $T$ are the transition probabilities for starting in a continuation state (target zone regime) and landing in another continuation state, $P_{cc}$, or starting in a continuation state and jumping to an abandonment state, $Q_{ca}$. The remaining $a$ rows are absorbing states.

The investors’ problem is to forecast the exchange rate given the initial state and threshold. The one step ahead forecast vector is

$$\hat{Y}_1 = TY = \begin{bmatrix} P_{cc} & Q_{ca} \\ 0 & I \end{bmatrix} \begin{bmatrix} Y_c \\ Y_a \end{bmatrix}.$$  

The first $c$ elements of the forecast vector are forecasts conditional on an initial state in the continuation region. For example, the first element of the forecast vector is

$$\hat{Y}_1(1) = E[y|y_i = v_1] = \sum_{j \in c} p_{1j}v_j + \sum_{j \in a} q_{1j}(w_j + d),$$

the expected value of the exchange rate next period conditional on being in state 1 this period. The forecast equals the value of the state variables in the continuation region weighted by the transition probabilities of moving from state 1 to a state in the continuation region plus the value of the state variables in the abandonment region weighted by the probabilities of moving from state 1 to a state in the abandonment region. The final $a$ elements of the vector are the random walk forecasts.

Multiperiod forecasts can be computed recursively, e.g.,

$$\hat{Y}_2 = T\hat{Y}_1 = T \begin{bmatrix} P_{cc} & Q_{ca} \\ 0 & I \end{bmatrix} \begin{bmatrix} Y_c \\ Y_a \end{bmatrix}.$$  

Step 3: The Bank’s problem: a modification of Nystrom’s method for stopping problems. The continuous state-space representation of the Bank’s dynamic stopping problem is

$$f(y_t) = \max[0, B - kA(y_t) + \beta Ef(y_{t+1}|y_t)]; \quad 0 < \beta < 1,$$
where the function \( f(y) \geq 0 \) is the option value of waiting. Here \( B \) represents the flow benefit and \( A(y) \) the agency cost of an incredible commitment. Dixit and Pindyck (1994, Appendix Chapter 4) show that the function is monotonically decreasing so there is a unique threshold, \( y^* \). I solve the Bank’s stopping problem with a modification of Nystrom’s method \(^{18}\) for solving integral equations.

Assume the threshold is known and partition the discrete state space into the continuation region and the stopping region. The discrete state-space representation of the Bank’s stopping problem at the maximum is

\[
\begin{bmatrix}
  f(Y_c) \\
  0
\end{bmatrix} = \begin{bmatrix}
  B + kA(Y_c) \\
  0
\end{bmatrix} + \beta \begin{bmatrix}
  P_{cc} & P_{cs} \\
  0 & I_{aa}
\end{bmatrix} \begin{bmatrix}
  f(Y_c) \\
  0
\end{bmatrix}.
\]

As in Nystrom’s method (evaluating the integral) solving for the unknown values, \( f(Y) \), just requires an inversion

\[
\begin{bmatrix}
  f(Y_c) \\
  0
\end{bmatrix} = 1/\beta \begin{bmatrix}
  P_{cc} & P_{cs} \\
  0 & I_{aa}
\end{bmatrix}^{-1} \begin{bmatrix}
  B + kA(Y_c) \\
  0
\end{bmatrix}.
\]

At the maximum the option value of waiting is non-negative in the continuation region and zero in the stopping region.

I solve the Bank’s problem by searching down the state space until I find the maximum. I start by assuming the threshold is state \( N - 1 \) and calculate the vector of option values. If the option value in the assumed threshold state is negative, \( f(Y_{N-1}) < 0 \), then I go to state \( N - 2 \) and repeat the process. When the option value evaluated at the assumed threshold is positive that is the maximum.

**Evaluating the agency cost.** The agency cost is a portion of expected depreciation due to the fact that the Bank cannot make a credible commitment. Under perfect credibility, the one period exchange rate forecast vector is

\[
\hat{\nu}_1 = PV
\]

and the \( k \) period forecast is

\[
\hat{\nu}_k = P^k V.
\]

Compute the agency cost for maturity \( j \) as

\[
a(Y, j) = (\{\hat{\nu}_j | d = 0\} - \hat{\nu}_j),
\]

the investors’ forecast of the exchange rate (with no fundamental currency disequilibrium) \(^{19}\) minus the perfect credibility forecast. I compute the utility loss from the agency cost as a linear function,

\[
A(y) = \sum_{j=1}^{J} \frac{1}{j} a(Y, j),
\]

where the weights decline with the forecast horizon.

---


\(^{19}\) The agency cost is due to a lack of credibility.
Step 4: Rational expectations equilibrium. In a rational expectations equilibrium investors’ conjectures about the Bank’s threshold are correct. Since expectations are endogenous in general there are multiple equilibria, e.g. see Fig. 3. The intuition for a brute force algorithm follows simple economic logic. Assume investors believe state \(k\) is the threshold. Solve the investors’ problem (step 2) to compute their expectations of the exchange rate (given their conjecture) and solve for the agency cost function given investors’ expectations. Now solve the Bank’s problem (step 3) to find the threshold, given agency cost. If state \(k\) is the threshold, then it is a rational expectations equilibrium. Solve the investors’ and Bank’s problem for all states \(y_k > 0\).

Appendix B. Sensitivity analysis

This appendix examines the robustness of the model solution to the particular parameterization I chose for Section 2. To examine the sensitivity of the results I perturb a parameter and solve the model. Three functions summarize the model results: the rational expectations equilibrium function, the agency currency premium function, and the interest rate differential function. This appendix shows plots of the three functions for the “baseline” parameterization of Section 2 and perturbations of the parameters. The functions are robust with respect to significant perturbations in the parameters. The rational expectations equilibria function retains the basic S shape, but its location and curvature changes. The number and location of imperfect credibility equilibria are sensitive to the parameterization. The agency currency premium function and the interest rate differential function are moderately sensitive to the location of the equilibrium threshold, but given a threshold they are not very sensitive to the parameterization. The graphs are titled B.\#x. The \# refers to function, 1 = the rational expectations equilibria function, 2 = the agency currency premium function, and 3 = the interest rate differential function. \(x\) refers to the parameter, \(a\) to the feedback coefficient, \(\sigma\) to the volatility, and \(d\) to the exogenous currency disequilibrium. The lines on the graph are labeled with \((\cdot|x=\text{value})\) to indicate the parameter value.

The exogenous environment

Exchange rate process. Two parameters characterize the exchange rate process in the exchange regime—the feedback coefficient \(a\), and the standard deviation of the shocks, \(\sigma\). The process in Section 2 is

\[
y_{t+1} = -ay_t + (1 - a)z_{t+1},
\]

\(z \sim t(0, \sigma^2, df)\),

\(a = 0.805\),

\(\sigma = 0.0053\),

\(df = 5\).

---

\(20\) My computer program uses more information about the structure of the problem and is more efficient.

\(21\) I used a coarser state space grid for these exercises, \(N = 25\).
In the first experiment I perturb the feedback coefficient by about 10%, $a \in \{0.705, 0.805, 0.905\}$. In the second I perturb the standard deviation by about 50%, $\sigma \in \{0.0025, 0.0053, 0.0075\}$. The figures below show the effect of the perturbation on three functions.

The feedback coefficient. Fig. B.1a shows the rational expectations equilibria function for perturbations in the feedback coefficient, $a$. The center line is the parameterization in Section 2.

The equilibria function for the smallest feedback coefficient, $a = 0.7$, is relatively flat and does not intersect the zero axis. Evidently when the feedback response is too small, there is not enough separation between the exchange regime and floating rates—there are no imperfect credibility equilibria. When the feedback coefficient is larger, the function has more curvature and two imperfect credibility equilibria exist.

Fig. B.2a shows the agency cost functions at the high confidence equilibria\(^{22}\) for quarter and 5-year investments.

The high confidence threshold equals 0.44% for either feedback coefficient, 0.8 or 0.9. The agency cost function is not very sensitive to changes in the feedback coefficient that do not affect the threshold.

Fig. B.3a shows the interest rate differential functions at the high confidence equilibria for quarter and 5-year investments.

\(^{22}\) I show agency costs for low confidence thresholds below.
Fig. B.2a. Agency currency premium.

Fig. B.3a. Interest rate differential.
The interest rate differential functions are not very sensitive to these changes in the feedback coefficient.

The standard deviation of the shock. Fig. B.1σ shows the rational expectations equilibria function for perturbations in the standard deviation of the shock, $\sigma$. The center line is the parameterization in Section 2.

When shocks are relatively small (the top line) there are no imperfect credibility equilibria. In the baseline case, two imperfect credibility equilibria exist. And when the shocks are larger only the low confidence imperfect credibility equilibrium exists.

Fig. B.2σ shows the agency cost function at the low confidence equilibrium for perturbations in the standard deviation of the shock, $\sigma$.

For the 5-year investment there is no difference in the agency cost. For the one-quarter investment, the average agency cost increases by a small amount with the volatility of the shock.

Fig. B.3σ shows the interest differential functions at the low confidence equilibrium for perturbations in the standard deviation of the shock, $\sigma$.

Substantially increasing the standard deviation of the shock slightly increases the interest rate differential. The increase, however, is not nearly enough to explain the increase in observed interest rate differentials in Hong Kong after July 1977.

Fundamental currency overvaluation. In this model fundamental currency overvaluation only affects the interest rate differential. Fig. B.d shows the interest rate differential function for $d \in \{0.0, 0.035, 0.07\}$ for quarter and 5-year investments at the high confidence equilibrium for the baseline parameter settings.
**Fig. B.2σ.** Agency currency premium.

**Fig. B.3σ.** Interest rate differential.
Fig. B.d. Interest rate differential.

Fig. B.k. Rational expectations equilibria: $f(y^*) = 0$. 
Increasing the fundamental currency overvaluation monotonically and dramatically increases the interest rate differential.

Tastes. Fig. B.k shows the effect of changing the utility function weight between the flow benefit and the agency cost.

With a small weight on the agency costs, the Bank never abandons and there is a unique full credibility equilibrium. Increasing the weight gives two imperfect credibility equilibria as the function shifts down and then only one.

References


