Money, labour supply, and growth in a liquidity costs economy*

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Introduction

This article examines the long-run consequences of the money supply growth rate on real variables and welfare by using an optimizing growth model with an endogenous labour supply and consumption liquidity costs, i.e. pecuniary transaction costs that affect consumption in the budget constraint of consumers.

This type of investigation has not been carried out before by the numerous studies on the issue of money superneutrality. Furthermore, a related purpose of the analysis of this paper is to make comparisons and study the equivalence with other frameworks (encompassing variable labour-leisure choices) used to investigate the relationship between "inflation and growth".

According to the liquidity costs approach, money makes the transactions necessary for consumption of physical goods easier. When labour decisions are exogenous, the consumption liquidity costs approach generates the same well-known Sidrauski (1967) results on money superneutrality as

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1 See, for example, Feenstra (1986) and Orphanides and Solow (1990).

2 Sidrauski's analysis is developed by entering real money balances into the utility function, considering infinitely-lived agents with a constant rate of time preference and a perfectly inelastic labour supply. Mo-
it is "functionally equivalent" to the introduction of money balances in the utility function.\footnote{This has been rigorously demonstrated by Feenstra (1986). However, from the money superneutrality point of view, the only basic difference between the money in the utility function approach and the liquidity costs approach is that higher long-run inflation also reduces consumption, in addition to real money balances, within the latter framework.}

The consideration of endogenous labour decisions in the standard optimizing monetary growth model leads to a violation of Sidrauski's results provided that consumption and leisure, on the one hand, and money, on the other, are not separable. The role of this hypothesis is studied firstly by Brock (1974).

Wang and Yip's (1992) contribution further develops the same issue by considering three alternative intertemporal optimizing models of capital accumulation and inflation with a variable labour supply. The three approaches under investigation are: the money in the utility function approach, the cash-in-advance approach, and the shopping-time approach.\footnote{Clear distinctions exist among consumption liquidity costs, cash-in-advance and shopping-time approaches. The cash-in-advance constraint approach states that money is necessary to acquire consumption goods and maybe investment goods (see Stockman (1981), Abel (1985) and Calvo (1986)). The shopping-time approach, introduced by Saving (1971) and the details of which are given in Kimbrough (1986) and Wang and Yip (1991), assumes that money allows the reduction of time spent in transactions and allows people to enjoy more leisure. Furthermore, another approach based on pecuniary liquidity costs is given by the production transaction costs approach, according to which money provides "shopping services" by freeing resources that in its absence would otherwise be necessary for production; see Dornbusch and Frenkel (1973) and Orphanides and Solow (1990).}

Wang and Yip's (1992) paper shows that anticipated inflation produces a negative effect on capital, labour and output, if some mild conditions on the functional forms used in the various approaches are imposed. Hence a sort of "qualitative equivalence" among the three different approaches in terms of the crucial comparative statics effects of inflation is obtained.

The key finding of our paper is that in the case of consumption liquidity costs combined with a variable labour supply, whether or not money is superneutral depends upon the class of the utility function considered. Monetary growth leaves capital and labour unaffected, when a constant relative risk aversion class of utility functions (with consumption and leisure Edgeworth dependent) is employed. If, instead, consumption and leisure are Edgeworth independent and at the same time preferences are iso-elastic in consumption, what matters in order to detect the final effects on capital and labour is the consumption intertemporal elasticity of substitution; if this elasticity is higher (lower) than one, steady-state inflation exerts a negative (positive) effect on capital and labour effort. When instead such an elasticity of substitution is equal to one, the superneutrality of money is again re-established. Finally, under CES preferences the crucial role for the final consequences of anticipated inflation is played by the intratemporal elasticity of substitution between consumption and leisure. Capital and
labour diminish, remain constant, or increase depending on whether this elasticity of substitution is greater, equal to, or less than one.

1 The model

Consider a monetary economy populated by identical agents who are infinitely-lived and have perfect foresight. The representative agent plays the double role of consumer and entrepreneur. There are two assets in the economy: physical capital and money. People hold money because it reduces transaction costs on consumption. Output is obtained by using labour, which is endogenously supplied, and capital, which is endogenously accumulated. The monetary authority expands the money supply stock at a given rate and lump-sum compensates private agents for the inflation tax.

The population size grows at a constant rate. The model is specified in continuous time. The representative agent decides on per capita consumption, $c$, labour effort, $l$, and saving by maximizing the following intertemporal utility function

$$\int_0^\infty U(c, l)e^{-\delta t}dt$$

subject to the flow budget constraint

$$c + \dot{m} + \dot{k} = f(k, l) + s - (n + \pi)m - nk - cz(m)$$

where $m$ represents per capita real money balances, $k$ is per capita capital stock, $s$ identifies per capita government transfers, $n$ is the population growth rate (exogenous), $\pi$ represents the inflation rate, $cz()$ gives per capita liquidity costs on consumption, and $\delta$ is the given rate of time preference.

The instantaneous utility function, $U( )$, defined over consumption and leisure, is increasing in consumption, but decreasing in labour, strictly concave, and twice-continuously differentiable. Both consumption and leisure are normal goods.

Per capita output is obtained by using capital and labour as inputs. The production function, $f( )$, is assumed to have the usual neoclassical properties of regularity and exhibit constant returns to scale. Linear homogeneity of $f( )$ assures that $f_{kl} > 0$, i.e. capital and labour are Edgeworth complements.

Saving can take the form of both money and capital accumulation. Total disposable income is given by output plus government transfers less the inflation tax on money holdings and per capita wealth times population growth. In addition, pecuniary liquidity costs on consumption must be subtracted. People must sacrifice some consumption for transaction purposes.
Real money balances can substitute the resources necessary for transactions, allowing for a reduction of the liquidity costs. The shopping cost for a unit of consumption is given by \( z \); this cost decreases by holding additional real money balances at a decreasing rate: \( z' < 0, z'' > 0 \). Total liquidity costs per capita are \( cz \).

The present value Hamiltonian for the dynamic optimization program of the representative agent is given by

\[
H = U(c, l) + \lambda \{ f(k, l) + s - (n + \pi)m - nk - c[1 + z(m)] \}
\]

where \( \lambda \) is the shadow value of wealth in the form of real money balances and physical capital. The control variables of the optimal dynamic problem for the representative agent are \( c \) and \( l \), the co-state variable is \( \lambda \), while the state variables are \( k \) and \( m \). The consumer-entrepreneur behaves competitively taking \( s \) and \( \pi \) as given.

The first order conditions for an interior solution are

\[
\begin{align*}
U_c(c, l) &= \lambda[1 + z(m)] & \text{(3a)} \\
U_l(c, l) &= -\lambda f_k(k, l) & \text{(3b)} \\
\dot{\lambda} - \lambda \delta &= \lambda[cz'(m) + n + \pi] & \text{(3c)} \\
\dot{\lambda} - \lambda \delta &= -\lambda[f_k(k, l) - n] & \text{(3d)}
\end{align*}
\]

together with the instantaneous budget constraint (2). In addition the usual transversality conditions must be respected

\[
\lim_{t \to \infty} \lambda me^{-\delta t} = \lim_{t \to \infty} \lambda ke^{-\delta t} = 0 \quad \text{(3e)}
\]

Conditions (3e) rule out explosive equilibria.

The first two equations are the familiar static efficiency conditions. According to equation (3a) the marginal utility of consumption must equal the marginal utility of wealth times the unit price of consumption, given by \( (1 + z) \). Equation (3b) asserts that the marginal utility of leisure must equal the marginal utility of wealth times the opportunity cost of a unit of leisure, i.e. the marginal product of labour.

Equations (3c) and (3d) are the intertemporal arbitrage relationships. They implicitly state that in equilibrium the rate of return on consumption, given by \( \delta - \lambda / \lambda \), has to be equal to the real return on each asset, which are given by \(-cz'(m) - n - \pi\), for money, and by \(f_k(k, l) - n\), for capital.

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5 While \( k \) is a predetermined variable, \( m \), as will become clear below, has, at general equilibrium level, a forward-looking nature because of the equilibrium condition on the money market.

6 Strict concavity of \( U() \) and \( f() \), and convexity of \( z() \) guarantee that the first order conditions for the optimum are necessary and sufficient and that the optimum is unique. See Arrow and Kurz (1970).
The nominal money supply is assumed to grow at a constant rate, given by $\theta$. Hence, the money market equilibrium requires per capita real money demand and supply to expand at the same rate

$$\frac{\dot{m}}{m} = \theta - \pi - n \quad (4)$$

Notice that $m$ is a forward-looking variable as the price level keeps changing in response to exogenous shocks to ensure that the equilibrium on the money market holds continuously.

Seignorage from money creation is distributed as lump-sum transfers to households:

$$s = \theta m \quad (5)$$

Finally, by adding up the representative agent’s budget constraint (2), the money market equilibrium condition (4), and the seignorage distribution scheme (5), we obtain the goods market equilibrium

$$f(k, l) = c[1 + z(m)] + \dot{k} + nk \quad (6)$$

Equation (6) states that the full employment output must be equal to the aggregate demand, given by gross consumption, i.e. $c(1 + z)$, plus total investment, i.e. $\dot{k} + nk$.

2 Long-run effects of inflation

Considering the steady-state equilibrium, where $\lambda = \dot{m} = \dot{k} = 0$, the model can be reduced to the following system

$$U_c(c, l) = \lambda[1 + z(\bar{m})] \quad (7a)$$

$$-\bar{c} U_c(c, l) = \frac{[f(k, l) - n\bar{k}]}{f_l(k, l)} \quad (7b)$$

$$-\bar{c}z'(\bar{m}) = \delta + \theta \quad (7c)$$

$$f_k(k, l) = \delta + n \quad (7d)$$

$$f(k, l) - n\bar{k} = \bar{c}[1 + z(\bar{m})] \quad (7e)$$

$$\bar{s} = \theta \bar{m} \quad (7f)$$

where the overbars denote long-run values and $\bar{\pi} = \theta - n$. 
The principal steady-state effects of inflation are described by the following basic multipliers\(^7\)

\[
\frac{d\bar{k}}{d\theta} = - \frac{\bar{c}z'f_{kl}\Omega}{\Delta}
\]

\[
\frac{d\left( \frac{\bar{k}}{l} \right)}{d\theta} = 0
\]

\[
\frac{d\bar{n}}{d\theta} = - \frac{[\Omega \Xi + f_{kl}(1 + z)\Gamma]}{\Delta} < 0
\]

\[
\frac{d\bar{c}}{d\theta} = - \frac{\bar{c}z'f_{kl}\Gamma}{\Delta} < 0
\]

where

\[
\Omega = f_1U_c[1 + \bar{c}U_{cc}/U_c - \bar{c}U_{ct}/U_t]
\]

\[
\Xi = \delta f_{kl} - f_1f_{kk} > 0
\]

\[
\Gamma = U_t(\delta + \bar{I}f_1/\bar{k}) + [f_1U_{et} + (1 + z)U_H]\bar{c}I/\bar{k} < 0
\]

\[
\Delta = \bar{c}f_{kl}\Gamma[z''(1 + z) - (z')^2] + \bar{c}z'\Omega \Xi
\]

The sign of \(\Delta\), important for the above multipliers, needs some discussion. In principle this sign can be either positive or negative. It is not difficult to show that \(\Delta\) is negative if the state matrix of the short-run dynamic model has a negative determinant. This is a requisite that must hold in order to ensure saddle-path stability of this perfect foresight model since the economy has two jump variables that respond instantaneously to new information, i.e. \(\lambda\) and \(m\), and one predetermined variable whose evolution is tied to the past, i.e. \(k\). Therefore with \(\Delta < 0\), the model can be saddle-point stable having two positive and one negative eigenvalue\(^9\).

Moreover, in order to fully ensure saddle-path stability, the trace of the state matrix must be positive, since only in this case are we sure that the dynamic system has two positive characteristic roots. Therefore only under the determinant and trace conditions does there exist a unique stable

\(^7\) Other relevant multipliers are

\[
\frac{d\bar{\lambda}}{d\theta} = -\{(U_{ct} - \bar{U}_{ct}\Omega(\bar{k}\Gamma))/f_1\} \frac{d\bar{c}}{d\theta}
\]

\[
\frac{ds}{d\theta} = \frac{\{[\Omega \Xi + f_{kl}(1 + z)](\bar{c}z'' - \theta) - \bar{m}f_{kl}\Gamma(z')^2\}}{\Delta}
\]

\(^8\) The expression \(z''(1 + z) - (z')^2\) appearing in \(\Delta\) is positive as a required condition of stability in the consumption shopping costs model with an inelastic labour supply.

\(^9\) Notice that the determinant condition for saddle-path stability, i.e. \(\Delta < 0\), ensures that anticipated inflation exerts a negative effect on real money demand. See Fischer (1979) and Wang and Yip (1992) on this economic implication of the stability condition.
(nonexplosive) solution that satisfies the necessary conditions of optimality and transversality\textsuperscript{10}.

Let us consider the long-run properties of the model. Capital-labour ratio is independent of the money growth rate since the production function is linearly homogeneous and the marginal productivity of capital remains constant (being determined by the given parameters $\delta$ and $n$). This implies that capital and labour change in the same direction — i.e. $\text{sgn}(d\bar{k}/d\theta) = \text{sgn}(d\bar{l}/d\theta)$ — and by the same proportion. Other clearly signed multipliers are those of real money balances and consumption, which both decline after the money growth rate is increased. In fact, as money holding is reduced, since higher inflation raises its opportunity costs, consumption diminishes as well, since it becomes more costly as its unit price, namely $1 + z$, is increased.

The steady-state effects of anticipated inflation on capital and labour are uncertain since they reflect the sign ambiguity of $\Omega$ which cannot be solved in general, but requires an explicit functional form for the utility function. In order to give a well-defined sign to $\Omega$, we focus on three special families of utility functions: i) a utility function belonging to the class of constant relative risk aversion preferences not additively separable in consumption and leisure (henceforth CRRA preferences); ii) a utility function which is iso-elastic in consumption but strongly separable in its arguments (henceforth QCRRA); and iii) a CES utility function\textsuperscript{11}.

Table 1 reports the qualitative effects of the long-run inflation on the principal variables for the different classes of preferences considered.

### 2.1 CRRA utility function

This utility function is defined as:

$$U = \frac{[c^\alpha(1-l)^\beta]^{1-\gamma}}{(1-\gamma)}, \quad \alpha, \beta, \gamma > 0, \quad \alpha + \beta, \quad \gamma \neq 1$$

(9)

For $\gamma = 1$, we have logarithmic preferences, $U = \alpha \log c + \beta \log(1 - l)$. Preferences (9) assume that consumption and leisure are Edgeworth dependent, provided that $\gamma \neq 1$.

With utility function (9), the supremeutrality of money prevails despite the endogeneity of the labour supply, since in this case it is not difficult

\textsuperscript{10} What would happen if $\Delta$ were positive? When $\Delta > 0$, we would have, if the trace were positive, three unstable roots so that there is no solution to the dynamic model, which is explosive; otherwise with a negative trace there is an infinite number of stable solutions (i.e. a globally stable economy). See Blanchard and Kahn (1980) and Buitert (1984).

\textsuperscript{11} These types of utility functions are chosen because they are often employed in similar studies (see, e.g., Fischer (1979), King-Plosser-Rebelo (1988), and Walsh (1998)) and can yield results in contrast to those of Wang-Yip (1992).
<table>
<thead>
<tr>
<th>Classes of utility functions</th>
<th>Effects of higher $\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRRA: $U = \frac{[e^{\alpha}(1-l)^{\beta}]^{1-\gamma}}{(1-\gamma)}$</td>
<td>$\frac{d\overline{k}}{d\theta} = 0, \frac{d\overline{l}}{d\theta} = 0, \frac{d(F)}{d\theta} = 0, \frac{d\bar{m}}{d\theta} = - \frac{d\bar{c}}{d\theta} = - \frac{d\bar{U}}{d\theta}$ if $\varepsilon \leq 1$</td>
</tr>
<tr>
<td>QCRRA: $U = \frac{e^{1-\varepsilon-1}}{(1-\varepsilon-1)} + V(l)$</td>
<td>$\text{sgn}(1-\varepsilon) \text{sgn}(1-\varepsilon) = 0, - - - \text{ if } \varepsilon \leq 1$</td>
</tr>
<tr>
<td>CES: $U = \left[ a \left( \frac{\sigma-1}{\sigma} \right) + b(1-l)(\frac{\sigma-1}{\sigma}) \right]^{\frac{\sigma}{\sigma-1}}$</td>
<td>$\text{sgn}(1-\sigma) \text{sgn}(1-\sigma) = 0, - - - \text{ if } \varepsilon \leq 1$</td>
</tr>
</tbody>
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Table 1: Principal comparative statics results: a qualitative synopsis

To ascertain that $\Omega = 0$ for any value of $\gamma$. Higher steady-state inflation leaves capital, labour and therefore output unaffected. This result sharply contrasts with the conclusions obtained by Wang and Yip (1992) where money balances enter the utility function: money is not supernormal since either a direct Tobin (1965) effect, in the case of the “asset substitution model”, or a reverse Tobin effect, in the case of the “transaction service model”, could occur. However, as previously noticed, in such a framework if some not very strong and plausible restrictions are imposed, the negative effect on capital and labour prevails, guaranteeing also the “qualitative equivalence” with other approaches.

The reason for our supernormality result is due to the fact that, when the utility function (9) is employed, the marginal rate of substitution between consumption and labour (i.e. $U_c/U_l$) is a hyperbole equilateral in consumption. Hence, the left hand side of equation (7b), i.e. $\delta U_c/U_l$, becomes independent of $\bar{c}$. Such an equation, along with the “modified golden rule”, determines the capital stock and working effort. Since both equations are independent of the money growth rate, inflation does not affect capital as well as labour, leading to the supernormality of money.

Moreover, $f(\bar{k}, \bar{l}) - n\bar{k}$ (that is, the output per capita net of investment necessary to maintain a fixed capital per capita) is constant, implying that the volume of liquidity costs also stays unaltered after the monetary shock. It follows from this fact that consumption must decrease so as to leave $c(1+z)$ unchanged, since the reduction of real money balances increases the unit shopping cost of consumption.

Welfare is unambiguously lowered by the higher money supply growth rate, because of the reduction in consumption. The effect of higher steady-

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12 Hence, this circumstance entails: $\frac{dk}{d\theta} = \frac{dl}{d\theta} = 0$. 
state inflation on the marginal utility of wealth is positive. Revenues from money creation can either decline or increase.

2.2 QCRRRA utility function

The second class of the utility function we study is of the type

\[ U = \frac{c^{1-\varepsilon^{-1}}}{(1-\varepsilon^{-1})} + V(l), \quad \varepsilon > 0, \quad \varepsilon \neq 1, \quad V' < 0, \quad V'' < 0 \]  

(10)

When \( \varepsilon = 1 \), utility function (10) collapses to \( U = \log c + V(l) \). \( \varepsilon \), that is the consumption intertemporal elasticity of substitution, plays a fundamental role in detecting the final effects of anticipated inflation on capital and labour, since \( \text{sgn}(\Omega) = \text{sgn}(\varepsilon - 1) \) in the expressions for \( d\bar{k}/d\theta \) and \( d\bar{l}/d\theta \).

A clear understanding of the model is obtained by plugging the QCRRRA utility function (10) into equation (7b) and by eliminating capital through the relationship \( \bar{k} = \bar{k}(\bar{l}) \) - obtained through the "modified golden rule" -; with \( \bar{k}' = \bar{l}'/\bar{k} > 0 \); we then get

\[ \bar{c} = [g(\bar{l})]^{1-\varepsilon^{-1}}, \quad g_t = -\frac{\bar{c}(1-z)}{\bar{f}_l} V'' - \frac{(\delta L_k + f_t)}{f_t} V' > 0 \]  

(a)

The elasticity of intertemporal substitution dictates the sign of the relationship between consumption and labour. Since consumption is unambiguously reduced, labour can therefore be increased, remain unchanged or reduced depending on whether \( \varepsilon \) is lower, equal to, or greater than one. Therefore when \( \varepsilon < (>)1 \), an increase in the monetary growth rate results in higher (lower) labour and capital, giving support to a direct (reverse) Tobin effect. Obviously, the particular case of \( \varepsilon = 1 \) brings us back to the situation obtained with preferences (9) : higher anticipated inflation does not change capital and labour.

This case clearly shows that under the consumption shopping costs approach, strong separability between consumption and leisure does not ensure per se the superneuutrality of money as with the money in the utility function approach (see Brock (1974)).

2.3 CES utility function

In this case we focus on the following functional form

\[ U = \left[ ac^{(\sigma-1)/\sigma} + b(1-l)^{(\sigma-1)/\sigma} \right]^{\sigma/\sigma-1}, \quad a, b > 1, \quad a + b = 1, \quad \sigma \geq 0 \]  

(11)
where $\sigma$ represents the intratemporal elasticity of substitution between consumption and leisure.

As for the QCRRA case, the consequences on labour (and hence capital) can be easily understood by employing the efficiency condition for the optimal choice of leisure and consumption. After eliminating capital stock through the "modified golden rule", we obtain

$$\bar{c} = (1 - \bar{l}) \left[ \frac{bh(\bar{l})}{a(1 - \bar{l})} \right]^\frac{\sigma - 1}{\sigma - 1}, \quad h_t = \frac{\delta \bar{I}}{f_I} + \frac{f_I}{f_I} > 0 \quad (b)$$

The relationship between consumption and labour is governed by the value of $\sigma$. If the elasticity of substitution between consumption and leisure is relatively low, i.e. $\sigma < 1$, the relationship between consumption and labour is negative. Therefore as higher inflation decreases consumption, leisure is reduced as well (since consumption and leisure are Edgeworth complements), implying higher labour effort and physical capital, and giving support to the Tobin effect. In fact, in this case the substitution effect of higher inflation is outweighed by the income effect; both effects derive from the reduced transactions costs on consumption. When $\sigma > 1$, as a positive link between consumption and labour occurs, the opposite results are obtained (since consumption and leisure become Edgeworth substitutes) leading to a reverse Tobin effect. In the case of the Cobb-Douglas preferences ($\sigma = 1$), the income and substitution effects exactly compensate each other leaving capital and labour unchanged. In this case equation (b) uniquely determines labour, which is therefore independent of the money supply growth rate, along with the capital stock given by the "modified golden rule".

3 Conclusions

This paper has examined the steady-state implications of anticipated inflation within an exogenous monetary growth model based on consumption liquidity costs and an endogenous labour supply.

The basic discovery of the paper is that the effects of the money supply growth rate upon capital and labour depend on the type of utility function chosen. However, whatever class of preferences are employed, the steady-state capital intensity remains unchanged, and consumption as well as real balances decline, whereas different results are obtained for capital and labour.

If a not-strongly separable CRRA utility function is considered, an increase in the rate of monetary expansion does not affect capital, labour and output, but only reduces consumption and real money balances.

When a strongly separable utility function is investigated, the steady-state consequences of inflation depend upon the intertemporal elasticity of
substitution of consumption: if it is lower (greater) than one, the traditional (reverse) Mundell-Tobin effect will prevail.

Furthermore, under CES preferences the final effects of inflation on capital, labour and output is governed by the intratemporal elasticity of substitution between consumption and leisure. When the degree of substitutability between consumption and leisure is relatively low (high), inflation increases (reduces) capital, labour and output. In the case of the Cobb-Douglas preferences the superneutrality of money prevails.

Our results differ from those derived through other approaches to money and growth -like the money in the utility function approach, the cash-in-advance approach, and the shopping-time approach- that consider an endogenous labour-leisure choice. Under mild restrictions on functional forms within these approaches a negative effect of anticipated inflation on capital and labour is detected.

We can conclude by saying that the conditions that make the consumption shopping costs approach "qualitatively equivalent" to the other approaches just mentioned are non-existent for a CRRA utility function, while they require that the elasticity of intertemporal substitution in the QCRRA case or the elasticity of intratemporal substitution in the CES case be greater than one.

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